Algebra 1 Review for Algebra 2

"Committed to Educating *Men for Others*
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Summer Reading

Each student taking Algebra 2 next Fall at Jesuit High School is responsible for reading this material and completing all of the Practice Problems at the end of each section. Each set of problems should be completed on a separate sheet of paper, showing all work. Bring the completed work to class at the beginning of the Fall semester.

Algebra 1 Review for Algebra 2

1. Solving Equations

This section requires that you solve an equation for a specific variable.

A. Procedure

1) Clear the equation of fractions by multiplying all the terms by the Lowest Common Denominator (LCD).
2) Isolate all terms that contain the variable you are solving for.
3) Combine like terms.
4) Factor, if necessary.
5) Divide to solve for the variable you are looking for.

B. Examples

1) Example 1: For the equation d = rt, solve for t:

$d = rt$

$\frac{d}{r} = \frac{rt}{r}$ Divide both sides by r

$\frac{d}{r} = t$ $\frac{r}{r} = 1$ on the right-hand side, leaving t

2) Example 2: For the equation S = rx + sx, solve for x:

$S = rx + sx$

$S = x(r + s)$ Factor the right side

$\frac{S}{r+s} = \frac{x(r+s)}{r+s}$ Divide both sides by (r + s)

$\frac{S}{r+s} = x$ $\frac{r+s}{r+s} = 1$ on the right-hand side, leaving x

3) Example 3: For the equation *p* = 2*l* + 2*w*, solve for *l*:

$p = 2l + 2w$

$p - 2w = 2l$ Subtract 2*w* from both sides

$\frac{p-2w}{2} = \frac{2l}{2}$ Divide both sides by 2

$\frac{p-2w}{2} = l$ $\frac{2}{2} = 1$ on the right-hand side, leaving *l*

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C. Practice Problems

1) Solve for t: $I = \Pr t$

2) Solve for p: $Q = \frac{p-q}{2}$

3) Solve for h: $A = \frac{1}{2}bh$

4) Solve for t: $A = at + bt$

5) Solve for π: $A = \pi r^2$

6) Solve for a: $S = \frac{1}{2}at^2$

7) Solve for l: $p = 2(l + w)$

8) Solve for h: $V = lwh$

9) Solve for n: $a = \frac{180(n-2)}{n}$

10) Solve for R: $C = K\frac{Rr}{R-r}$

2. Straight Line Graphs

This review of straight line graphs will refresh your Algebra 1 skills on this particular area and assist you in meeting the challenges of a variety of new topics that you will encounter in Algebra 2.

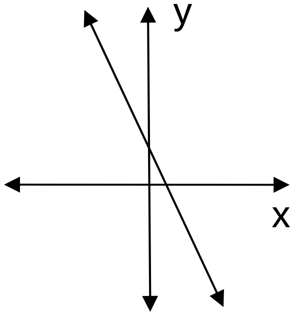
A. Slope-Intercept Form of an Equation: y = mx + b

1) Example 1. Graph y = -2X + 1

a) Create a table of ordered pairs:

x	y	(x,y)
0	1	(0,1)
1	-1	(1,-1)
2	-3	(2,-3)

b) Plot points:



2) Example 2. Graph a) y = x, b) y = 2x, c) y = 4x on the same set of axes.

a)

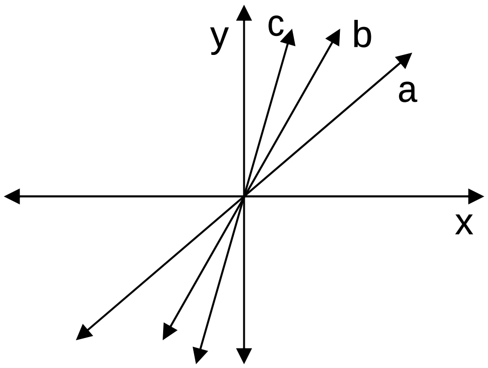
x	y	(x,y)
-1	-1	(-1,-1)
0	0	(0,0)
1	1	(1,1)
2	2	(2,2)

b)

x	y	(x,y)
-1	-2	(-1,-2)
0	0	(0,0)
1	2	(1,2)
2	4	(2,4)

c)

x	y	(x,y)
-1	-4	(-1,-4)
0	0	(0,0)
1	4	(1,4)
2	8	(2,8)



Note that the bigger the number in front of the x (the coefficient of x), the steeper the graph. The number in front of x tells us the slope of the graph.

3) Example 3. Graph a) y = -2x, b) y = -2x + 3, c) y = -2x + 3 on the same set of axes.

a)

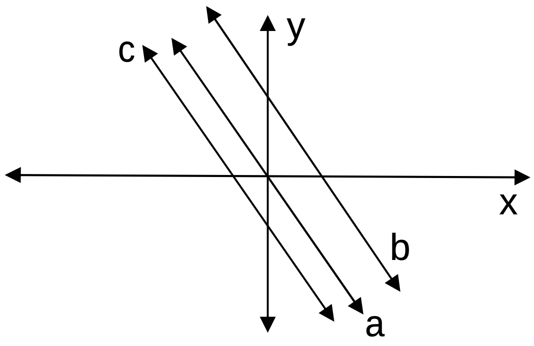
x	y	(x,y)
-1	2	(-1,2)
0	0	(0,0)
1	-2	(1,-2)
2	-4	(2,-4)

b)

x	y	(x,y)
-1	5	(-1,5)
0	3	(0,3)
1	1	(1,1)
2	-1	(2,-1)

c)

x	y	(x,y)
-1	1	(-1,1)
0	-1	(0,-1)
1	-3	(1,-3)
2	-5	(2,-5)

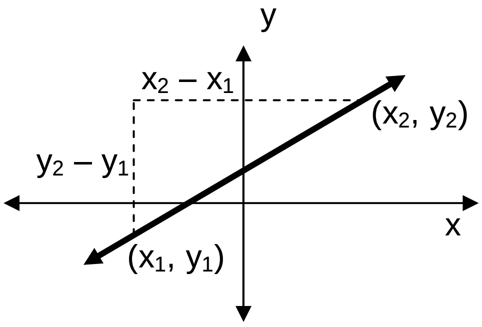


Note that all these lines are **parallel** as they have the **same slope**. They cross at different points on the y-axis. When a straight line is in the form y = mx + b, then the "m" stands for the slope and the "b" stands for the y-intercept.

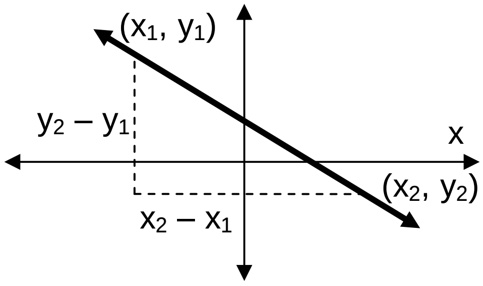
4) Note also that when the slope is negative, the lines go down to the right (decreases), and when the slope is positive, the lines go up to the right (increases).

5) Definition of slope. SLOPE = $\frac{rise}{run} = \frac{change\ \ in\ y}{change\ \ in\ x} = \frac{y_2 - y_1}{x_2 - x_1} = m$

Positive slope:



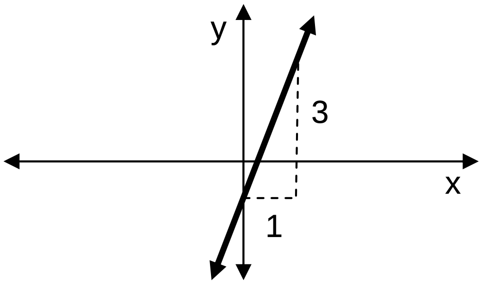
Negative slope:



6) Example 4. Find slope and y-intercept of the following and graph each equation.

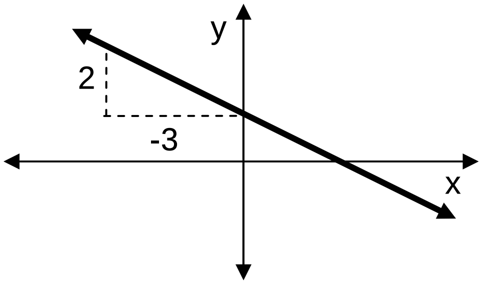
a) y = 3x – 1

Slope = 3
y-intercept = -1
$\frac{rise}{run} = \frac{3}{1}$



b) 3y + 2x = 6

3y = -2x + 6
y = $-\frac{2}{3}x + 2$
Slope = -2/3
y-intercept = 2
$\frac{rise}{run} = \frac{-2}{3}$



B. The Point-Slope Equation

$y - y_1 = m(x - x_1)$ where m = slope and (x_1, y_1) is a point on the line.

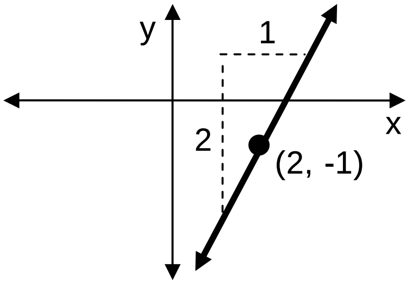
1) Example 1. Write the point-slope equation of a line with slope -1/2 containing the point (3, -1):

$$y - y_1 = m(x - x_1)$$
$$y + 1 = -\frac{1}{2}(x - 3)$$

2) Example 2. Write the slope-intercept equation of a line with slope 3 containing the point (-2, 5):

$$y - y_1 = m(x - x_1)$$
$$y - 5 = 3(x + 2)$$
$$y - 5 = 3x + 6$$
$$y = 3x + 11$$

3) Example 3. Graph the line with slope 2 that passes through (2, -1):

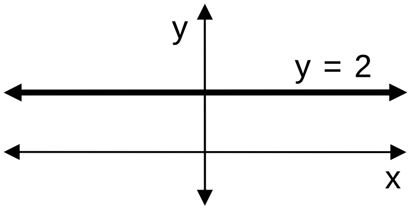


4) Example 4. Find the slope of the line containing the points (-2, 1) and (3, -4):

Slope = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-1}{3-(-2)} = \frac{-5}{5} = -1$

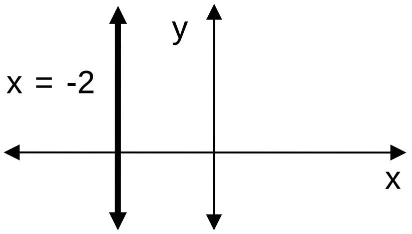
5) Example 5. Find the slope of the line y = 2.

In Slope-Intercept format, y = 0x +2. Therefore, m = 0, or the slope is zero. A line with zero slope is always horizontal.



6) Example 6. Find the slope of line x = -2.

For any value of y, x will always equal -2. Therefore, the line must always be vertical. The change in x is zero, making the slope undefined since we cannot divide a number by zero (Slope equation).



C. Graphing Using Intercepts

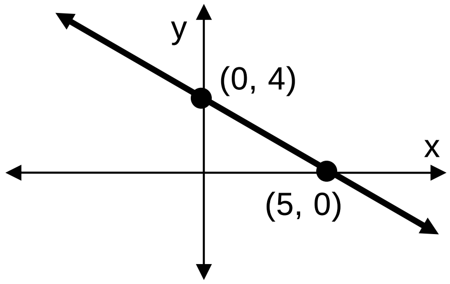
1) Use this method when the equation is in the general form: Ax + By = C

2) General steps:

   a) Find the x-intercept by setting y=0
   b) Find the y-intercept by setting x=0
   c) Plot the x-intercept and y-intercept on the axes
   d) Draw the line by connecting the two points

3) Example. Plot the equation 4x + 5y = 20.

   a) When x = 0, 5y = 20, therefore y = 4. The y- intercept is (0, 4)
   b) When y = 0, 4x = 20, therefore x = 5. The x-intercept is (5, 0)
   c) Plot these points and join them together with a straight line.



D. Perpendicular Lines

1) Two lines are perpendicular when their slopes are the negative reciprocal of one another.
2) If two negative reciprocals are multiplied by each other, the result is -1.
3) The reciprocal of x is 1/x and the negative reciprocal of x is -1/x.
4) So if one slope is *m*, then the other slope is -1/m.
5) Example: if the slopes of two lines are -2/3 & 3/2, they are perpendicular.
6) Example: The slope of a line perpendicular to a line with slope 4 is -1/4.

E. Practice Problems

1) For the following, create a table of ordered pairs and plot the points. Draw straight lines on the same axes:
   a) y = 3x   b) y = 3x - 2   c) y = 3x + 1
   What do you notice about these lines?

2) For the following, create a table of ordered pairs and plot the points. Draw straight lines on the same axes:
   a) y = -x   b) y = -x – 1   c) y = -x +3
   What do you notice about these lines?

3) Find the slope and y-intercept, then graph the following using the slope and intercept:
   a) y = 2x + 4   b) y = -4x – 1   c) y = $\frac{1}{3}x+2$

4) Rewrite the following in slope-intercept form and find the slope and y-intercept:
   a) 2x + 3y = 9   b) 2y – 5x = 8   c) 6y – x = 5

5) Write slope-intercept form of the line with given slope m and a point on the line:
   a) m=3; (4, 1)   b) $m=\frac{1}{2}$; (2, -4)   c) m= -3; (-4, -6)

6) Graph the following straight lines on the same set of axes:
   a) m=2; point on line (2, -5)   b) $m=-\frac{1}{2}$; point on line (-1, -2)
   What do you notice about these lines?

7) Find the slope of the line containing the following points:
   a) (3, 1) & (-2, 4)   (b) (-4,-5) & (2, 1)   c) (-3, -1) & (7, -1)

8) Graph each line and state the slope of each line:
   a) y = -5   b) x = 2   c) y = 3   d) x = 0

9) Graph the following given two points on the line and find the slope of each line.
a) (1, -4) & (-2, 3) b) (2, 3) & (2, -3) c) (5, -2) & (-1, -1)

10) Graph the following using the x and y intercepts:
a) 2x + 7y = 14 b) 5x – 3y = 15 c) 2x – 3y = 4

11) Write slope-intercept form of line parallel to given line & through given point.
a) y= 2x + 4; (3,1) b) y = -3x -1; (-2, 3) c) y = 0.5x -10; (4, 6)

12) Write slope-intercept form of line perpendicular to given line and through a given point.
a) y= 2x - 3; (2,4) b) y = -3x -4; (-6, 3) c) y = 0.5x -8; (1, 3)

3. **<u>Factoring Quadratic Trinomials</u>** (also known as the "Bottoms Up" method)

When you have a trinomial whose coefficient is other than "1" use the following method to factor:

A. Example 1

1) $3x^2+10x-8$ Original equation
2) $x^2+10x-24$ Multiply the constant by the leading coefficient.
3) $(x+12)(x-2)$ Factor the trinomial as if the leading coefficient were "1"
4) $(x+\frac{12}{3})(x-\frac{2}{3})$ Divide the constant by the original coefficient
5) $(x+4)(x-\frac{2}{3})$ Simplify each fraction
6) $(x+4)(3x-2)$ Move any denominator for each binomial into the lead coefficient position.
7) Check the resulting factors by using FOIL.

B. Example 2

1) $6y^2-19y+10$
2) $y^2-19y+60$
3) $(y-15)(y-4)$
4) $(y-\frac{15}{6})(y-\frac{4}{6})$
5) $(y-\frac{5}{2})(y-\frac{2}{3})$
6) $(2y-5)(3y-2)$

Check by using FOIL:

$(2y-5)(3y-2)$

$6y^2-4y-15y+10$

$6y^2-19y+10$ Checks!

C. Example 3

1) $4x^2-13x+10$
2) $x^2-13x+40$
3) $(x-8)(x-5)$
4) $(x-\frac{8}{4})(x-\frac{5}{4})$
5) $(x-2)(x-\frac{5}{4})$
6) $(x-2)(4x-5)$

Don't forget to check

D. Example 4

1) $3+35a-12a^2$
2) $-(12a^2-35a-3)$

Factor out -1

3) $-(a^2-35a-36)$
4) $-(a-36)(a+1)$
5) $-(a-\frac{36}{12})(a+\frac{1}{12})$
6) $-(a-3)(12a+1)$ or $(3-a)(12a+1)$

Don't forget to check

E. Example 5

1) $6x^3-15x-x^2$
2) $x(6x^2-x-15)$

Correct descending order

3) $x(x^2-x-90)$
4) $x(x-10)(x+9)$
5) $x(x-\frac{10}{6})(x+\frac{9}{6})$
6) $x(x-\frac{5}{3})(x+\frac{3}{2})$
7) $x(3x-5)(2x+3)$

Check!

4. <u>Factoring Polynomials</u>

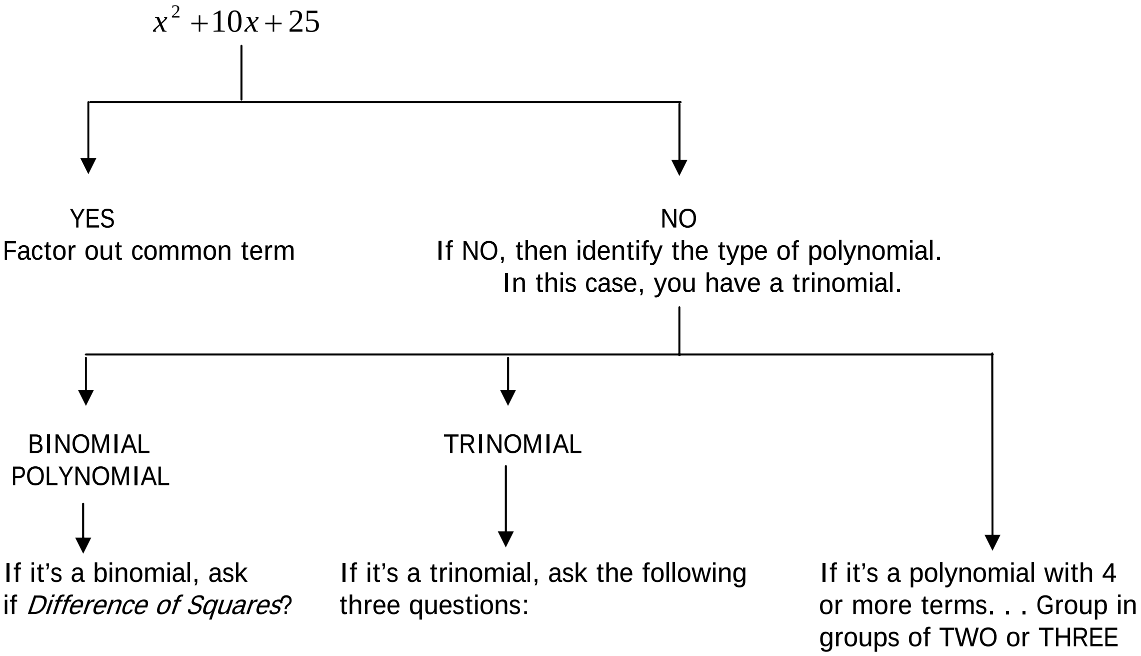
A. Overview

Factoring is the opposite procedure of distributing. Factoring a polynomial is the same as *finding an equivalent expression expressed as a product.*

Example:	$2x+4$	Binomial
	$2(x+2)$	Factored (expressed as a product)

B. General Procedure

Factoring polynomials can follow a set pattern. The diagram below is a helpful schematic to follow when factoring. Always start any factoring problem with the question: *"Are there common factors in each term?"*



$x^2+10x+25$

YES
Factor out common term

NO
If NO, then identify the type of polynomial.
In this case, you have a trinomial.

BINOMIAL POLYNOMIAL

If it's a binomial, ask if *Difference of Squares*?

TRINOMIAL

If it's a trinomial, ask the following three questions:

If it's a polynomial with 4 or more terms. . . Group in groups of TWO or THREE

1. Is the expression a PERFECT SQUARE TRINOMIAL?
2. Is the expression a TRINOMIAL with a leading coefficient of 1?
3. Is the expression a TRINOMIAL with a leading coefficient greater than 1?

In this case you have a perfect square trinomial

1. $x^2+10x+25$
2. $(x+5)(x+5)$

"What two factors of +25 add up to +10?"
Answer: "+5 and +5"

C. Binomials

1) Overview

a) *Binomials* have two terms. When factoring binomials, always start by asking "Are there any common factors in each term?" If so, then factor out the common term and start over with the resulting binomial. Look for a *Difference of Squares* when you start over (explained below).

b) Example:

$4x^2-16$

$4(x^2-4)$

$4(x+2)(x-2)$

The resulting binomial is a *difference of squares* and can be factored further

2) Difference of Squares

a) Once you have factored out a common term, the other characteristic to look for is a *Difference of Squares*. A *Difference of Squares* is recognized exactly according to its name: A DIFFERENCE (subtraction) of two SQUARES.

b) Example: Difference of Squares $x^2 - 25$

x^2 and 25 are both squares

c) Example: NOT a Difference of Squares $x^2 + 25$

Not a difference of squares because there is a + sign

d) If the problem is a *Difference of Squares*, then write out two parentheses with opposite signs and the square root of each perfect square.

e) Example: $x^2 - 25$ *Difference of Squares* because x^2 is a square and 25 is a square with a minus sign between them.

$(x+5)(x-5)$ Use FOIL to check

3) Practice Problems – Factor each of the following binomials. Make sure you follow the procedure outlined above. Write out each step. Remember how to multiply decimals and fractions.

a) $x^2 - 49$
b) $4x^2 - 1$
c) $2x^4 - 50x^2$
d) $x^2 - y^2$
e) $9x^2 - y^2z^2$
f) $.25a^4 - .36b^4$
g) $\left(\frac{x}{2}\right)^2 - \left(\frac{y}{2}\right)^2$
h) $x^8 - y^8$

HINT: Make sure you keep factoring any resulting difference of squares

D. Trinomials

1) Definitions

Linear Term

Quadratic term

Constant

$$x^2 + 10x + 25$$

2) Overview

a) Trinomials have three terms. Start factoring trinomials the same way you factor binomials. Ask: “Are there any common factors in all three terms? If yes, factor out the common term. If no, then ask yourself one of the following three questions:

- Is the trinomial a PERFECT SQUARE trinomial?
- Does the trinomial have a leading coefficient of 1?
- Does the trinomial have a leading coefficient greater than 1?

b) Each of these questions will lead you to a different method of factoring.

3) Is the Trinomial a Perfect Square Trinomial?

a) A perfect square trinomial such as $x^2+10x+25$ has a constant which is a perfect square and a linear term that is a sum of the perfect square factors.

b) Example 1:

$x^2+10x+25$ Ask yourself: “What factors of +25 add up to the coefficient of the linear term? (In this case, +10)

$(x+5)(x+5)$

c) Example 2

$x^2-12x+36$ Q. “What factors of +36 add up to -12?”
$(x-6)(x-6)$ A. (-6)(-6) = +36 and (-6)+(-6) = -12

4) Does the Trinomial have a leading coefficient of “1” ?

a) Use the same procedure as a perfect square trinomial. Ask the same question:

b) Example:

$x^2-7x+10$ Q. “What factors of +10 add up to -7?”
$(x-5)(x-2)$ A. (-2)(-5) = 10 and (-2)+(-5) = -7

5) Does the Trinomial have a leading coefficient greater than “1” ?

When you have a trinomial whose coefficient is other than “1” use the “bottoms up” method to factor:

a) Example 1:

$3x^2+10x-8$	Multiply the constant by the leading coefficient.
$x^2+10x-24$	Factor as if the leading coefficient were “1”
$(x+12)(x-2)$	Divide the constant by the original coefficient
$(x+\frac{12}{3})(x-\frac{2}{3})$	Simplify each fraction
$(x+4)(x-\frac{2}{3})$	Move any denominator for each binomial into the lead coefficient position.
$(x+4)(3x-2)$	Check the resulting factors by using FOIL.

b) Example 2:

$6y^2-19y+10$

$y^2-19y+60$

$(y-15)(y-4)$

$(y-\frac{15}{6})(y-\frac{4}{6})$

$(y-\frac{5}{2})(y-\frac{2}{3})$

$(2y-5)(3y-2)$

Check by using FOIL

$(2y-5)(3y-2)$

$6y^2-4y-15y+10$

$6y^2-19y+10$ Checks!

c) Example 3:

$4x^2-13x+10$

$x^2-13x+40$

$(x-8)(x-5)$

$(x-\frac{8}{4})(x-\frac{5}{4})$

$(x-2)(x-\frac{5}{4})$

$(x-2)(4x-5)$

Don’t forget to CHECK

d) Example 4:

$3+35a-12a^2$

$-(12a^2-35a-3)$

Factor out -1

$-(a^2-35a-36)$

$-(a-36)(a+1)$

$-(a-\frac{36}{12})(a+\frac{1}{12})$

$-(a-3)(12a+1)$ or $(3-a)(12a+1)$

e) Example 5:

$6x^3-15x-x^2$

$x(6x^2-x-15)$

Correct descending order

$x(x^2-x-90)$

$x(x-10)(x+9)$

$x(x-\frac{10}{6})(x+\frac{9}{6})$

$x(x-\frac{5}{3})(x+\frac{3}{2})$

$x(3x-5)(2x+3)$

CHECK

6) Practice Problems

Factor each of the following trinomials according to the procedure above. Highlight which problems are *perfect square* trinomials. Also notice which trinomials use the “bottoms up” method. Write out each step. Box your answer. Try to recognize a pattern in analyzing the problems.

a) $x^2+10x+25$

b) $x^2+7x+10$

c) $6m^2+23m+20$

d) $2x^2-10x+8$

e) $6z^2+5z-6$

f) $a^4+5a^2b+6b^2$

g) $12x^2+23x+10$

h) $(xy)^2+4xy+4$

i) x^3-x^2-30x

j) $3x^2+10x-8$

k) $2x^2+34x-220$

l) $x^2-2xy-48y^2$

m) $(x+1)^2+2(x+1)+1$ HINT: Treat the (x + 1) as a “single” variable; try substituting “y” for “x+1” and solve

n) x^6+2x^3-15 HINT: Replace x^6 with υ^2 and x^3 with υ^1 so you can rewrite the problem as $\upsilon^2+2\upsilon-15$. After factoring, replace the υ with x^3 and rewrite.

o) $16y^2+49+56y$ Hint: Correct descending order?

E. Polynomials

Polynomials of this type have four (or more) terms. When factoring these types of polynomials, group the polynomial using groups of 2 terms and 2 terms, or 3 terms and 1 term. Certain characteristics will lead you to group correctly. The examples below illustrate this idea.

1) Groups of 2 terms and 2 terms

a) Example 1:

$x^2+xy+2x+2y$	Since the terms are all connected by addition signs, you may move terms in any order. In this case, group the first two and last two terms.
$\left(x^2+xy\right)+\left(2x+2y\right)$	From each new binomial, factor out what is common in each term.
$x(x+y)+2(x+y)$	Notice that the resulting factor is the same.
$(x+2)(x+y)$	The resulting sum is really just a large binomial. Factor out the (x + y) and add the leftover factors.

b) Example 2:

x^2-y^2-x-y	Group the first two terms and the last two terms.
$(x^2-y^2)-(x+y)$	Remember to change the sign in front of the last "y" because when you distribute the negative sign, the last "y" becomes positive.
$(x+y)(x-y)-1(x+y)$	Factor out the (x - y) and subtract the resulting factors.
$(x+y)(x-y-1)$	

c) Example 3:

$3x-3+2x-2$	Group the first two and the last two terms.
$(3x-3)+(2x-2)$	Factor.
$3(x-1)+2(x-1)$	Factor (x - 1) and add the resulting terms.
$(x-1)(3+2)$ or $5(x-1)$	

d) Example 4:

$a^3-ab^2-2a^2+2b^2$	Group the first two and the last two terms.
$(a^3-ab^2)-(2a^2-2b^2)$	Change the sign in front of the last $2b^2$
$a(a^2-b^2)-2(a^2-b^2)$	Factor out common terms.
$(a^2-b^2)(a-2)$	Note that a^2-b^2 is a difference of squares.
$(a-b)(a+b)(a-2)$	Factor.

2) Groups of Three Terms and One Term

Instead of grouping a four-term polynomial by groups of two, sometimes you must group the first three terms and the last term. You group the first three terms if you notice the following characteristics:

- Your polynomial contains THREE perfect squares $x^2 + xy + y^2 - 1$. Remember "1" is a perfect square
- The last perfect square is subtracted.
- The "middle" term is the product of the two factors on either side of it.

a) Example 1:

$x^2 + xy + y^2 - 1$	Group first three terms and last term.
$(x^2 + xy + y^2) - 1$	The first three terms are a perfect square trinomial. Factor.
$(x + y)^2 - 1$	After you factor, you have a *Difference of Squares* binomial.
$[(x + y) + 1][(x + y) - 1]$	Polynomials are all "additions," erase the parentheses.
$(x + y + 1)(x + y - 1)$	Check by FOIL

b) Example 2:

$x^2 + 10x + 25 - y^2$	The first three terms are a perfect square trinomial. The last term is a subtracted perfect square. Group "three" and "one."
$(x^2 + 10x + 25) - y^2$	Factor the perfect square trinomial.
$(x + 5)^2 - y^2$	Factor the binomial.
$[(x + 5) - y][(x + 5) + y]$	Drop the parentheses.
$(x + 5 - y)(x + 5 + y)$	Check by FOIL

3) Practice Problems

Factor each of the following polynomials. Look for the characteristics which make each one a grouping of "2 and 2" or a grouping of "3 and 1." Show every step according to the examples above. Label each step. Box your answer.

a) $xy - xz - y^2 + yz$

b) $x^3 + 3x^2 - 4x - 12$

c) $x^2 + 6x + 9 - y^2$

d) $x^2 - y^2 + 8y - 25$

e) $x^2 - 12x + 36 - 49$

5. Exponential Notation

A. Definitions and rules

1) Exponential notation is x^y, where *x* is the base and *y* is the exponent.

2) Example: 10^5 – 10 is the base and 5 is the exponent, and the expression is equivalent to 10 x 10 x 10 x 10 x 10, or 10 five times.

3) Rules for order of operations:
   a) Calculate within the innermost grouping symbol
   b) Simplify all exponential expressions
   c) Perform all multiplication and division, working from left to right
   d) Perform all addition and subtraction, working from left to right

4) Examples:
   a) $(3 \bullet 4)^2 = (12)^2 = 144$
   b) $3 \bullet 4^2 = 3 \bullet 16 = 48$

B. Multiplying Powers with like bases

1) *Product rule*: for any number *a* and any positive integers *m* and *n*, $a^m \bullet a^n = a^{m+n}$. Examples:

   a) $b^4 \bullet b^3 = b^7$
   b) $x^2 \bullet x^9 = x^{11}$
   c) $(r+s)^7 \bullet (r+s)^6 = (r+s)^{13}$

2) *Quotient rule*: for any number *a* and any positive integers *m* and *n*, $\frac{a^m}{a^n} = a^{m-n}$

   Examples:

   a) $\frac{x^8}{x^2} = x^{8-2} = x^6$

   b) $\frac{7^9}{7^4} = 7^{9-4} = 7^5$

   c) $\frac{p^5q^7}{p^2q} = \frac{p^5}{p^2} \bullet \frac{q^7}{q} = p^{5-2}q^{7-1} = p^3q^6$

C. Zero as an Exponent

1) The quotient rule can be used to determine what *0* should mean when it appears as an exponent. For any nonzero value of *a*, $a^0 = 1$.

2) Example: $\frac{a^4}{a^4} = 1$ and $\frac{a^4}{a^4} = a^{4-4} = a^0$, or $\frac{a^4}{a^4} = a^0 = 1$

D. Raising a power to a power

1) Power rule: for any number *a* and any whole numbers *m* and *n*, $(a^m)^n = a^{mn}$.

2) To raise a power, multiply the exponents and leave the base unchanged.

3) Example: $(m^2)^5 = m^{2\bullet 5} = m^{10}$

4) Example: $(3^5)^4 = 3^{5\bullet 4} = 3^{20}$

E. Raising a product to a power

1) For any numbers *a* and *b* and any whole number *n*, $(ab)^n = a^n b^n$

2) To raise a product to a power, raise each factor to that power.

3) Example: $(4a)^3 = 4^3 a^3 = 64a^3$

4) Example: $(a^7 b)^2 = a^{14} b^2$

F. Raising a quotient to a power

1) For any real numbers *a* and *b*, with *b* not equal to 0, and any whole number *n*, $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

2) To raise a quotient to a power, raise the numerator to the power and divide by the denominator to the same power.

3) Example: $\left(\frac{x}{5}\right)^2 = \frac{x^2}{5^2} = \frac{x^2}{25}$

4) Example: $\left(\frac{5}{a^4}\right)^3 = \frac{5^3}{\left(a^4\right)^3} = \frac{125}{a^{12}}$

G. Practice Problems: simplify

1) $r^4 \bullet r^6$

2) $(3y)^4(3y)^8$

3) $s^4 \bullet s^5 \bullet s^2$

4) $\left(\frac{3}{x}\right)^4$

5) $(-2a)^3$

6) $\left(\frac{4a^2b}{3c^7}\right)^3$

7) $\left(\frac{5a^7}{2b^5c}\right)^0$

8) $\left(x^3y\right)^2\left(x^2y^5\right)$

6. Radical Expressions and Equations

A. Square roots

1) The number *c* is a square root of *a* if $c^2 = a$.

2) Examples:

25	its square roots are 5 and -5
16	its square roots are 4 and -4
81	its square roots are 9 and -9

3) The non-negative root of a number is called the *principle square root* of that number. A radical sign is generally used when finding the square roots and indicates the principle root. Thus, $\sqrt{25} = 5$, not -5, and $\sqrt{225} = 15$ and $-\sqrt{64} = -8$.

B. Multiplying roots

1) The *Product Rule for Square Roots*: for any real numbers $\sqrt{A}$ and $\sqrt{B}$, $\sqrt{A} \bullet \sqrt{B} = \sqrt{AB}$

2) To multiply square roots, multiply the radicands and take the square roots.

3) Example: $\sqrt{5} \bullet \sqrt{7} = \sqrt{5 \bullet 7} = \sqrt{35}$

4) Example: $\sqrt{\frac{2}{3}} \bullet \sqrt{\frac{7}{5}} = \sqrt{\frac{2 \bullet 7}{3 \bullet 5}} = \sqrt{\frac{14}{15}}$

C. Simplifying and factoring

1) A radical expression of a square root is simplified when its radicand has no factor other than 1 that is a perfect square. Use the product rule in reverse, $\sqrt{AB} = \sqrt{A} \bullet \sqrt{B}$.

2) Example: $\sqrt{48} = \sqrt{16} \bullet \sqrt{3} = 4\sqrt{3}$

3) Example: $\sqrt{50} = \sqrt{25} \bullet \sqrt{2} = 5\sqrt{2}$

4) Example: $\sqrt{a^2 b} = \sqrt{a^2} \bullet \sqrt{b} = a\sqrt{b}$

D. Simplifying square roots of powers

1) To take the square root of an even power such as x^{10}, note that $x^{10} = \left(x^5\right)^2$, then $\sqrt{x^{10}} = \sqrt{\left(x^5\right)^2} = x^5$.

2) Example: $\sqrt{p^6} = \sqrt{\left(p^3\right)^2} = p^3$

3) Example: $\sqrt{t^{22}} = \sqrt{\left(t^{11}\right)^2} = t^{11}$

4) The exponent of the square root is half the exponent of the radicand.

5) If a radicand has an odd power, simplify by factoring: $\sqrt{x^9} = \sqrt{x^8 \bullet x} = \sqrt{x^8}\sqrt{x} = x^4\sqrt{x}$

E. Dividing radical expressions

1) *Quotient Rule for Square Roots*: for any real numbers $\sqrt{A}$ and $\sqrt{B}$, with *B* not equal to 0, $\frac{\sqrt{A}}{\sqrt{B}} = \sqrt{\frac{A}{B}}$.

2) To divide two square roots, divide the radicands and take the square root.

3) Example: $\frac{\sqrt{27}}{\sqrt{3}} = \sqrt{\frac{27}{3}} = \sqrt{9} = 3$

4) Example: $\sqrt{\frac{14}{50}} = \frac{\sqrt{14}}{\sqrt{50}} = \frac{\sqrt{7 \bullet 2}}{\sqrt{25 \bullet 2}} = \frac{\sqrt{7} \bullet \sqrt{2}}{\sqrt{25} \bullet \sqrt{2}} = \frac{\sqrt{7}}{\sqrt{25}} = \frac{\sqrt{7}}{5}$

F. Adding and subtracting radical expressions

1) The sum of a rational number and an irrational number, such as $5+\sqrt{2}$, cannot be simplified. However, the sum of like radicals that have a common radical factor can be simplified.

2) Example: $9\sqrt{17}-3\sqrt{17}=(9-3)\sqrt{17}=6\sqrt{17}$

3) Example: $5\sqrt{2}-\sqrt{18}=5\sqrt{2}-3\sqrt{2}=2\sqrt{2}$

G. Solving radical equations

1) Step 1: isolate a radical term
2) Step 2: use the principle of squaring
3) Step 3: Solve the new equation

H. The principle of squaring

1) If a=b, then $a^2=b^2$

2) Example: $\sqrt{x}+3=7$
$\sqrt{x}+3-3=7-3$ (subtract 3 from both sides)
$\sqrt{x}=4$
$\left(\sqrt{x}\right)^2=4^2$ (square both sides)
$x=16$

I. Practice problems: simplify

1) $\sqrt{361}$

2) $\frac{\sqrt{72}}{\sqrt{2}}$

3) $\sqrt{50}$

4) $\sqrt{25a^2}$

5) $\sqrt{\frac{4x^3}{50x}}$

6) $\sqrt{90}+4\sqrt{40}$

7) $\sqrt{\frac{6}{5x}}$ rationalize the denominator

8) $\sqrt{7a^3b}\sqrt{21a^2b^3}$

9) $\sqrt{40}$

10) Solve for x: $\sqrt{10x}+7=15$