Algebra 2 Review for Pre-Calculus

"Committed to Educating *Men for Others*
through Academic Excellence,
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Summer Reading

Each student taking Pre-Calculus next Fall at Jesuit High School is responsible for reading this material and completing all of the Practice Problems at the end of each section. Each set of problems should be completed on a separate sheet of paper, showing all work. Bring the completed work to class at the beginning of the Fall semester.

Algebra 2 Review for Pre-Calculus

1. Complex Numbers

A. Complex numbers are numbers such as 3i and 2-5i. These numbers have a real component (*a*) and an imaginary component (*bi*) and are written in *a+bi* form.

B. Complex numbers are equal if the real components are equal and the imaginary components are equal.

Example: $5x+8yi=10+4i$ if x=2 and y=1/2

C. Complex numbers can be added or subtracted by combining the real parts and their imaginary parts.

1) Example: (7+2i) + (3-6i) = (7+3) + (2-6)i = 10-4i

2) Example: (12+5i) – (7+2i) = (12-7) + (5-2)i = 5+3i

D. Complex numbers can be multiplied using the FOIL method

Example: (5+3i)(7-2i) = (5*7) + (5*-2i) +(3i*7) + (3i*-2i)
= 35 – 10i + 21i – 6i^2
= 35 + 11i + 6
= 41 + 11i

E. Complex numbers can be divided by using the conjugate.

1) (3+4i) has a conjugate (3-4i)

2) If a complex number appears as a denominator in a fraction, the numerator and denominator must be multiplied by the conjugate of the denominator in order to rewrite it in the standard form "*a + bi*."

3) Example:

$$\frac{2+9i}{3-4i}\times\frac{3+4i}{3+4i}=\frac{(2\times3)+(2\times4i)+(9i\times3)+(9i\times4i)}{(3\times3)+(3\times4i)+(-4i\times3)+(-4i\times4i)}$$

$$=\frac{6+8i+12i+36i^2}{9+12i-12i-16i^2}=\frac{-30+20i}{25}=\frac{-6}{5}+\frac{4i}{5}$$

F. Solving equations involving complex numbers:

1) Example: using the quadratic formula. Solve for x: x^2+4x+6

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \bullet 1 \bullet 6}}{2 \bullet 1} = \frac{-4 \pm \sqrt{16 - 24}}{2} = \frac{-4 \pm \sqrt{-8}}{2} = \frac{-4 \pm 2i\sqrt{2}}{2} = 2 \pm i\sqrt{2}$$

2) Powers of i: $i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$, $i^5 = i$ and the pattern continues

Example: $i^{45} = i^{44} \times i = \left(i^4\right)^{11} \times i = (1)^{11} \times i = i$

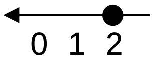
2. Inequalities

A. Inequalities are statements that two quantities are not equal.

1) To solve the inequality means to find all the solutions to the statement.
2) Most inequalities have an infinite number of solutions.
3) Solving an inequality is similar to solving an equation.
4) Both sides of the inequality can be added to or subtracted from.
5) Multiplying or dividing an inequality by a negative number reverses the inequality sign.
6) Example: solve for x:

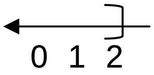
$$\begin{aligned} &5x - 2 \le -2x + 12 \\ &7x - 2 \le 12 \\ &7x \le 14 \\ &x \le 2 \end{aligned}$$

7) Notation:



0 1 2

or



0 1 2

8) Interval notation: $(-\infty, 2]$

9) Example: solve $\frac{5}{x-6} \ge 0$

Since the numerator is positive, the fraction will be positive whenever the denominator is positive: therefore $x - 6 \ge 0$ and $x \ge 6$ or $[6, \infty)$

B. Inequalities and absolute value expressions.

1) Properties of absolute value:

|X| <2 means -2 < X < 2
|X| >2 means X< -2 or X > 2
|X+1| < 4 means the distance between X and -1 is less than 4 units

2) Example: solve for x:

$|X+1| < 4$

$-4 < X+1 < 4$

$-5 < X < 3$

Subtracted 1 from all terms

Interval notation: (-5,3)

3) Example: solve for x:

$|X-2| \geq 3$

$X-2 \geq 3$ or $-(X-2) \geq 3$

Reverse inequality when divide by -1

$X \geq 5$ or $X-2 \leq -3$ (divide by -1)

$X \leq -1$

Interval notation: $(-\infty,-1] \cup [5,\infty)$

C. Solving a quadratic inequality – make one side equal to 0 and factor**.**

1) Example:

$$5x^2 + x \geq 6$$

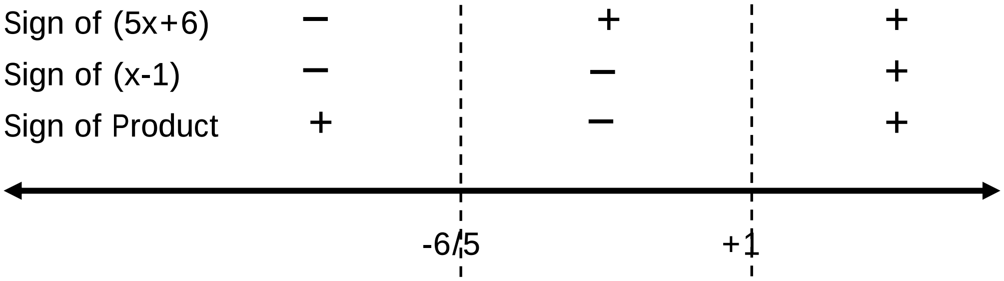
$$5x^2 + x - 6 \geq 0$$

$$(5x+6)(x-1) \geq 0$$

a) The factors are zero at -6/5 and +1

b) Divide the number line into intervals and make a sign chart:

Sign of (5x+6)	−	+	+
Sign of (x-1)	−	−	+
Sign of Product	+	−	+



c) The solution is the set of values for which the product is positive. Test the end points of the interval to make sure they are included in the solution set.

d) Interval notation: $\left(-\infty,-\frac{6}{5}\right] \cup [1,\infty)$

2) Example:

$$\frac{5x+2}{x-1} \geq 3$$

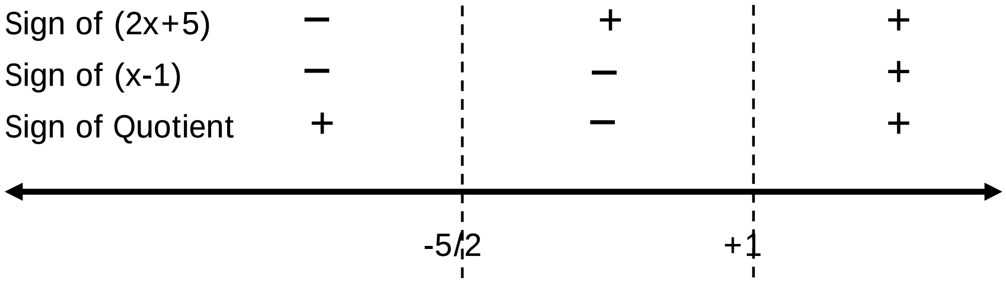
$$\frac{5x+2}{x-1} - 3 \geq 0$$

$$\frac{5x+2}{x-1} - \frac{3(x-1)}{(x-1)} \geq 0$$

$$\frac{5x+2-3x+3}{x-1} \geq 0$$

$$\frac{2x+5}{x-1} \geq 0$$

a) Solution: 2x+5 = 0 when x = -5/2
x-1 = 0 when x = 1
Make a sign chart for the factors
Test the intervals and end points
The solution is the part of the number line where the quotient is positive.



b) Test the end points: at x = -5/2, the quotient is positive, so -5/2 is in the solution set. At x = 1, the quotient is undefined, so 1 is not in the solution set.

c) Therefore, the answer is $\left(-\infty,-5/2\right]\cup(1,\infty)$

D. Practice Problems

1) Simplify the expressions

a) $(7+2i)+(3-4i)$

b) $\left(2-\sqrt{-36}\right)\left(7-\sqrt{-81}\right)$

c) $(-6+i)-(4-2i)$

d) Find *x* and *y*: $8x+i(2y)=3+4i$

e) $(5+6i)^2$

f) i^{93}

g) $i(2-3i)^2$

h) $\dfrac{-2-i}{-3+i}$

i) $\dfrac{5+i}{-2+3i}$

2) Solve the equations

a) $3x^2+23=5$

b) $x^3+27=0$

c) $5x^2+2x+4=0$

d) $x^3+5x^2+4x=0$

e) $x^2=9x-20$

f) $x^2+9=0$

3) Solve the inequalities. Express answer in interval notation.

a) $3x+2<5x-9$

b) $|7x+3| \geq -2$

c) $1 \leq \frac{2x+1}{3} \leq 5$

d) $\frac{4}{x^2-4} < 0$

e) $|X| < 6$

f) $\frac{(x+3)(x-4)}{(x+1)} \geq 0$

g) $x(3x+4) \geq 5$

h) $\frac{5}{x+2} \geq \frac{3}{x-4}$

i) $9x^2-4 < 0$

j) $\frac{x}{3x+1} < \frac{2}{x+6}$

k) $|5x-2| \leq 2$

3. Integer Exponents

In the expression a^n, *a* is called the base and *n* is called the exponent. The base can be anything from a number to a variable or an algebraic expression.

A. When the exponent is a positive integer, a^n tells us to multiply *a* by itself *n* times. Examples:

1) $5^4 = 5 \bullet 5 \bullet 5 \bullet 5 = 645$

2) $\left(\frac{1}{2}\right)^2 = \frac{1}{2} \bullet \frac{1}{2} = \frac{1}{4}$

3) $(-3)^4 = (-3)(-3)(-3)(-3) = 81$

4) $-2(3)^4 = -2 \bullet 3 \bullet 3 \bullet 3 \bullet 3 = 162$

Note the difference between the last two exponentials. The exponent only applies to what ever immediately precedes it.

B. When the exponent is zero, the result will always be equal to one. Examples:

1) $5^0 = 1$

2) $(-2)^0 = 1$

3) $(3x+1)^0 = 1$

C. When the exponent is a negative number, the result is the reciprocal of the positive exponent. Examples:

1) $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

2) $(-3)^{-5} = \frac{1}{(-3)^5} = \frac{1}{-243}$

D. Laws of Exponents

1) $a^n a^m = a^{n+m}$ Example: $2^3 \bullet 2^4 = 2^{3+4} = 2^7$

2) $(a^m)^n = a^{mn}$ Example: $(3^3)^2 = 3^{2\bullet 3} = 3^6$

3) $(ab)^n = a^n b^n$ Example: $(20)^3 = (2 \bullet 10)^3 = 2^3 \bullet 10^3 = 8 \bullet 1000 = 8000$

4) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ Example: $\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5} = \frac{32}{243}$

5) $\frac{a^m}{a^n} = a^{m-n}$ Example: $\frac{2^5}{2^7} = 2^{5-7} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

4. Radical Notation

The expression $\sqrt[n]{a}$ is called the principal *n*th root of *a*. This expression is called a *radical*, the number "*a*" is called the *radicand*, and the number *n* is called the index of the radical.

If *n* = 2, we do not write the index. The principal *n*th root is defined as follows:

Let "n" be a positive integer greater than 1 and let "a" be a real number.

A. Rules and examples:

1) if *a* = 0, then $\sqrt[n]{a} = 0$

2) if *a* > 0, then $\sqrt[n]{a}$ is the *positive* real number *b* such that $b^n = a$

3) if *a* < 0 and *n* is odd, then $\sqrt[n]{a}$ is the *negative* real number b such that $b^n = a$

4) if a < 0 and *n* is even, then $\sqrt[n]{a}$ is not a real number

5) $\sqrt{16} = 4$

6) $\sqrt[5]{\frac{1}{32}} = \frac{1}{2}$

7) $\sqrt[3]{-8} = -2$

8) $\sqrt[4]{-16}$ *is not a real number*

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B. Properties and laws of radicals with positive integer index

1) $(\sqrt[n]{a})^n = a$, if $\sqrt[n]{a}$ is a real number Example: $\left(\sqrt[3]{-8}\right)^3 = -8$
2) $\sqrt[n]{a^n} = a$, if $a \geq 0$ Example: $\sqrt{5^2} = 5$
3) $\sqrt[n]{a^n} = a$, if a < 0 and n is odd Example: $\sqrt[3]{(-2)^3} = -2$
4) $\sqrt[n]{a^n} = |a|$, if a < 0 and n is even Example: $\sqrt[4]{(-2)^4} = |-2| = 2$

Note that $\sqrt[4]{(-2)^4} = 2$, but $(\sqrt[4]{-2})^4$ is not a real number since $\sqrt[4]{-2}$ is not a real number. The following laws are true as long as each root is a real number:

1) $\sqrt[n]{ab} = \left(\sqrt[n]{a}\right)\left(\sqrt[n]{b}\right)$
2) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
3) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

C. Simplifying radical expressions

1) To simplify a radical expression with index *n*, we look for factors of the radicand that can be written with an exponent of *n*.

To simplify $\sqrt[3]{320}$, we look for any factors of 320 that are perfect cubes:

$$\sqrt[3]{320} = \sqrt[3]{64 \bullet 5} = \sqrt[3]{64} \bullet \sqrt[3]{5} = 4\sqrt[3]{5}$$

Another example is:

$$\sqrt[3]{16x^3y^8z^4} = \sqrt[3]{8 \bullet 2 \bullet x^3 \bullet y^6 \bullet y^2 \bullet z^3 \bullet z} = \sqrt[3]{8x^3y^6z^3} \bullet \sqrt[3]{2y^2z} = 2xy^2z\sqrt[3]{2y^2z}$$

2) If the denominator of a fraction contains an nth root radical, to simplify the radical, we need to rationalize the denominator. This process is done by multiplying the numerator and denominator of the fraction by an nth root radical to end up with an nth power in the radicand.

Examples:

a) $\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \bullet \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{\sqrt{5^2}} = \frac{\sqrt{5}}{5}$

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b) $\dfrac{1}{\sqrt[3]{x}} = \dfrac{1}{\sqrt[3]{x}} \bullet \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^3}} = \dfrac{\sqrt[3]{x^2}}{x}$

3) If an expression has an exponent that is a rational number but is not an integer, we can use radicals to define it.

a) $x^{1/3} = \sqrt[3]{x}$

b) $x^{3/5} = \left(\sqrt[3]{x}\right)^5 = \left(\sqrt[3]{x^5}\right)$

c) $125^{2/3} = \left(\sqrt[3]{125}\right)^2 = 5^2 = 25$

D. Practice Problems

Simplify the following, and rationalize the denominator when appropriate. Your answers should have no negative or rational exponents.

1) $\left(-3x^{-2}\right)\left(4x^4\right)$

2) $\left(-3a^2b^{-5}\right)^3$

3) $\dfrac{\left(3y^3\right)\left(2y^2\right)^2}{\left(y^4\right)^3}$

4) $\left(\dfrac{x^6}{9y^{-4}}\right)^{-1/2}$

5) $a^{4/3}a^{-3/2}a^{1/6}$

6) $\sqrt[3]{8a^6b^{-3}}$

7) $\sqrt{\dfrac{3x}{2y^3}}$

8) $\sqrt[3]{\dfrac{2x^4y^4}{9x}}$