Algebra 2 Review for Pre-Calculus

"Committed to Educating Men for Others
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Summer Reading

Each student taking Pre-Calculus next Fall at Jesuit High School is responsible for reading this material and completing all of the Practice Problems at the end of each section. Each set of problems should be completed on a separate sheet of paper, showing all work. Bring the completed work to class at the beginning of the Fall semester.

Algebra 2 Review for Pre-Calculus

1. Complex Numbers

A. Complex numbers are numbers such as 3i and 2-5i. These numbers have a real component (*a*) and an imaginary component (*bi*) and are written in *a+bi* form.

B. Complex numbers are equal if the real components are equal and the imaginary components are equal.

Example: $5x+8yi=10+4i$ if x=2 and y=1/2

C. Complex numbers can be added or subtracted by combining the real parts and their imaginary parts.

1) Example: (7+2i) + (3-6i) = (7+3) + (2-6)i = 10-4i

2) Example: (12+5i) – (7+2i) = (12-7) + (5-2)i = 5+3i

D. Complex numbers can be multiplied using the FOIL method

Example: (5+3i)(7-2i) = (5*7) + (5*-2i) +(3i*7) + (3i*-2i)
= 35 – 10i + 21i – 6i^2
= 35 + 11i + 6
= 41 + 11i

E. Complex numbers can be divided by using the conjugate.

1) (3+4i) has a conjugate (3-4i)

2) If a complex number appears as a denominator in a fraction, the numerator and denominator must be multiplied by the conjugate of the denominator in order to rewrite it in the standard form "*a + bi*."

3) Example:

$$\frac{2+9i}{3-4i}\times\frac{3+4i}{3+4i}=\frac{(2\times3)+(2\times4i)+(9i\times3)+(9i\times4i)}{(3\times3)+(3\times4i)+(-4i\times3)+(-4i\times4i)}$$

$$=\frac{6+8i+12i+36i^2}{9+12i-12i-16i^2}=\frac{-30+20i}{25}=\frac{-6}{5}+\frac{4i}{5}$$

F. Solving equations involving complex numbers:

1) Example: using the quadratic formula. Solve for x: $x^2 + 4x + 6$

Quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \bullet 1 \bullet 6}}{2 \bullet 1} = \frac{-4 \pm \sqrt{16-24}}{2} = \frac{-4 \pm \sqrt{-8}}{2} = \frac{-4 \pm 2i\sqrt{2}}{2} = 2 \pm i\sqrt{2}$$

2) Powers of i: $i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$, $i^5 = i$ and the pattern continues

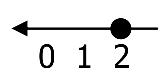
Example: $i^{45} = i^{44} \times i = \left(i^4\right)^{11} \times i = (1)^{11} \times i = i$

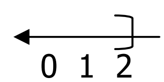
2. <u>Inequalities</u>

A. Inequalities are statements that two quantities are not equal.

1) To solve the inequality means to find all the solutions to the statement.
2) Most inequalities have an infinite number of solutions.
3) Solving an inequality is similar to solving an equation.
4) Both sides of the inequality can be added to or subtracted from.
5) Multiplying or dividing an inequality by a negative number reverses the inequality sign.
6) Example: solve for x:

$$5x - 2 \le -2x + 12$$
$$7x - 2 \le 12$$
$$7x \le 14$$
$$x \le 2$$

7) Notation: 

or 

8) Interval notation: $(-\infty, 2]$

9) Example: solve $\dfrac{5}{x-6} \ge 0$

Since the numerator is positive, the fraction will be positive whenever the denominator is positive: therefore $x - 6 \ge 0$ and $x \ge 6$ or $[6, \infty)$

B. Inequalities and absolute value expressions.

1) Properties of absolute value:

$|X| < 2$ means $-2 < X < 2$
$|X| > 2$ means $X < -2$ or $X > 2$
$|X+1| < 4$ means the distance between X and -1 is less than 4 units

2) Example: solve for x:

$|X+1| < 4$
$-4 < X+1 < 4$
$-5 < X < 3$ — Subtracted 1 from all terms
Interval notation: $(-5,3)$

3) Example: solve for x:

$|X-2| \geq 3$
$X-2 \geq 3$ or $-(X-2) \geq 3$
$X \geq 5$ or $X-2 \leq -3$ (divide by -1) — Reverse inequality when divide by -1
$X \leq -1$
Interval notation: $(-\infty,-1] \cup [5,\infty)$

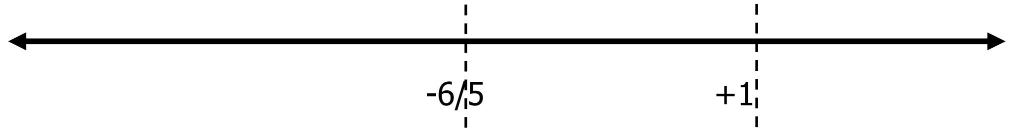
C. Solving a quadratic inequality – make one side equal to 0 and factor.

1) Example:

$$5x^2 + x \geq 6$$
$$5x^2 + x - 6 \geq 0$$
$$(5x+6)(x-1) \geq 0$$

a) The factors are zero at -6/5 and +1
b) Divide the number line into intervals and make a sign chart:

Sign of (5x+6)	−	+	+
Sign of (x-1)	−	−	+
Sign of Product	+	−	+



c) The solution is the set of values for which the product is positive. Test the end points of the interval to make sure they are included in the solution set.

d) Interval notation: $\left(-\infty,-\frac{6}{5}\right] \cup [1,\infty)$

2) Example: $\dfrac{5x+2}{x-1} \geq 3$

$$\frac{5x+2}{x-1} - 3 \geq 0$$

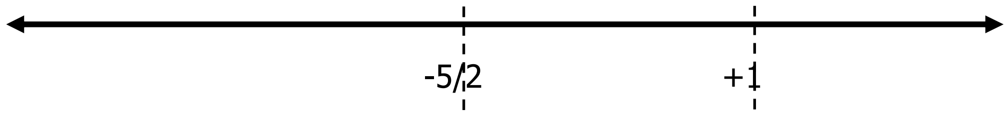
$$\frac{5x+2}{x-1} - \frac{3(x-1)}{(x-1)} \geq 0$$

$$\frac{5x+2-3x+3}{x-1} \geq 0$$

$$\frac{2x+5}{x-1} \geq 0$$

a) Solution: 2x+5 = 0 when x = -5/2
x-1 = 0 when x = 1
Make a sign chart for the factors
Test the intervals and end points
The solution is the part of the number line where the quotient is positive.

Sign of (2x+5)	−	+	+
Sign of (x-1)	−	−	+
Sign of Quotient	+	−	+



b) Test the end points: at x = -5/2, the quotient is positive, so -5/2 is in the solution set. At x = 1, the quotient is undefined, so 1 is not in the solution set.

c) Therefore, the answer is $\left(-\infty, -\frac{5}{2}\right] \cup (1, \infty)$

D. Practice Problems

1) Simplify the expressions

a) $(7+2i)+(3-4i)$

b) $(2-\sqrt{-36})(7-\sqrt{-81})$

c) $(-6+i)-(4-2i)$

d) Find x and y: $8x+i(2y)=3+4i$

e) $(5+6i)^2$

f) i^{93}

g) $i(2-3i)^2$

h) $\dfrac{-2-i}{-3+i}$

i) $\dfrac{5+i}{-2+3i}$

2) Solve the equations

a) $3x^2+23=5$

b) $x^3+27=0$

c) $5x^2+2x+4=0$

d) $x^3+5x^2+4x=0$

e) $x^2=9x-20$

f) $x^2+9=0$

3) Solve the inequalities. Express answer in interval notation.

a) $3x+2<5x-9$

b) $|7x+3| \geq -2$

c) $1 \leq \dfrac{2x+1}{3} \leq 5$

d) $\dfrac{4}{x^2-4} < 0$

e) $|X| < 6$

f) $\dfrac{(x+3)(x-4)}{(x+1)} \geq 0$

g) $x(3x+4) \geq 5$

h) $\dfrac{5}{x+2} \geq \dfrac{3}{x-4}$

i) $9x^2-4 < 0$

j) $\dfrac{x}{3x+1} < \dfrac{2}{x+6}$

k) $|5x-2| \leq 2$

3. Integer Exponents

In the expression a^n, *a* is called the base and *n* is called the exponent. The base can be anything from a number to a variable or an algebraic expression.

A. When the exponent is a positive integer, a^n tells us to multiply *a* by itself *n* times. Examples:

1) $5^4 = 5 \bullet 5 \bullet 5 \bullet 5 = 645$
2) $\left(\frac{1}{2}\right)^2 = \frac{1}{2} \bullet \frac{1}{2} = \frac{1}{4}$
3) $(-3)^4 = (-3)(-3)(-3)(-3) = 81$
4) $-2(3)^4 = -2 \bullet 3 \bullet 3 \bullet 3 \bullet 3 = 162$

Note the difference between the last two exponentials. The exponent only applies to what ever immediately precedes it.

B. When the exponent is zero, the result will always be equal to one. Examples:

1) $5^0 = 1$
2) $(-2)^0 = 1$
3) $(3x+1)^0 = 1$

C. When the exponent is a negative number, the result is the reciprocal of the positive exponent. Examples:

1) $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
2) $(-3)^{-5} = \frac{1}{(-3)^5} = \frac{1}{-243}$

D. Laws of Exponents

1) $a^n a^m = a^{n+m}$ Example: $2^3 \bullet 2^4 = 2^{3+4} = 2^7$
2) $(a^m)^n = a^{mn}$ Example: $(3^3)^2 = 3^{2\bullet 3} = 3^6$
3) $(ab)^n = a^n b^n$ Example: $(20)^3 = (2 \bullet 10)^3 = 2^3 \bullet 10^3 = 8 \bullet 1000 = 8000$
4) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ Example: $\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5} = \frac{32}{243}$
5) $\frac{a^m}{a^n} = a^{m-n}$ Example: $\frac{2^5}{2^7} = 2^{5-7} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

4. Radical Notation

The expression $\sqrt[n]{a}$ is called the principal *n*th root of *a*. This expression is called a *radical*, the number "*a*" is called the *radicand*, and the number *n* is called the index of the radical.

If *n* = 2, we do not write the index. The principal *n*th root is defined as follows:

Let "n" be a positive integer greater than 1 and let "a" be a real number.

A. Rules and examples:

1) if *a* = 0, then $\sqrt[n]{a} = 0$
2) if *a* > 0, then $\sqrt[n]{a}$ is the *positive* real number *b* such that $b^n = a$
3) if *a* < 0 and *n* is odd, then $\sqrt[n]{a}$ is the *negative* real number b such that $b^n = a$
4) if a < 0 and *n* is even, then $\sqrt[n]{a}$ is not a real number
5) $\sqrt{16} = 4$
6) $\sqrt[5]{\dfrac{1}{32}} = \dfrac{1}{2}$
7) $\sqrt[3]{-8} = -2$
8) $\sqrt[4]{-16}$ *is not a real number*

B. Properties and laws of radicals with positive integer index

1) $(\sqrt[n]{a})^n = a$, if $\sqrt[n]{a}$ is a real number Example: $\left(\sqrt[3]{-8}\right)^3 = -8$
2) $\sqrt[n]{a^n} = a$, if *a* ≥ 0 Example: $\sqrt{5^2} = 5$
3) $\sqrt[n]{a^n} = a$, if a < 0 and *n* is odd Example: $\sqrt[3]{(-2)^3} = -2$
4) $\sqrt[n]{a^n} = |a|$, if a < 0 and *n* is even Example: $\sqrt[4]{(-2)^4} = |-2| = 2$

Note that $\sqrt[4]{(-2)^4} = 2$, but $(\sqrt[4]{-2})^4$ is not a real number since $\sqrt[4]{-2}$ is not a real number. The following laws are true as long as each root is a real number:

1) $\sqrt[n]{ab} = \left(\sqrt[n]{a}\right)\left(\sqrt[n]{b}\right)$
2) $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$
3) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

C. Simplifying radical expressions

1) To simplify a radical expression with index *n*, we look for factors of the radicand that can be written with an exponent of *n*.

To simplify $\sqrt[3]{320}$, we look for any factors of 320 that are perfect cubes:

$$\sqrt[3]{320} = \sqrt[3]{64 \bullet 5} = \sqrt[3]{64} \bullet \sqrt[3]{5} = 4\sqrt[3]{5}$$

Another example is:

$$\sqrt[3]{16x^3y^8z^4} = \sqrt[3]{8 \bullet 2 \bullet x^3 \bullet y^6 \bullet y^2 \bullet z^3 \bullet z} = \sqrt[3]{8x^3y^6z^3} \bullet \sqrt[3]{2y^2z} = 2xy^2z\sqrt[3]{2y^2z}$$

2) If the denominator of a fraction contains an nth root radical, to simplify the radical, we need to rationalize the denominator. This process is done by multiplying the numerator and denominator of the fraction by an nth root radical to end up with an nth power in the radicand.

Examples:

a) $\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \bullet \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{\sqrt{5^2}} = \frac{\sqrt{5}}{5}$

b) $\frac{1}{\sqrt[3]{x}} = \frac{1}{\sqrt[3]{x}} \bullet \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^3}} = \frac{\sqrt[3]{x^2}}{x}$

3) If an expression has an exponent that is a rational number but is not an integer, we can use radicals to define it.

a) $x^{1/3} = \sqrt[3]{x}$

b) $x^{3/5} = \left(\sqrt[3]{x}\right)^5 = \left(\sqrt[3]{x^5}\right)$

c) $125^{2/3} = \left(\sqrt[3]{125}\right)^2 = 5^2 = 25$

D. Practice Problems

Simplify the following, and rationalize the denominator when appropriate. Your answers should have no negative or rational exponents.

1) $\left(-3x^{-2}\right)\left(4x^{4}\right)$

2) $\left(-3a^{2}b^{-5}\right)^{3}$

3) $\dfrac{\left(3y^{3}\right)\left(2y^{2}\right)^{2}}{\left(y^{4}\right)^{3}}$

4) $\left(\dfrac{x^{6}}{9y^{-4}}\right)^{-1/2}$

5) $a^{4/3}a^{-3/2}a^{1/6}$

6) $\sqrt[3]{8a^{6}b^{-3}}$

7) $\sqrt{\dfrac{3x}{2y^{3}}}$

8) $\sqrt[3]{\dfrac{2x^{4}y^{4}}{9x}}$

5. Algebraic Expressions

A polynomial in x is a sum of terms in the form:

$$a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$$

Where n is a non-negative integer and each coefficient a_k is a real number, $a_n \neq 0$. a_n is the lead coefficient of the polynomial and n is the degree of the polynomial.

$3x^{4} + 5x^{3} + (-7x) + 4$	has a lead coefficient of 3 and degree 4
$x^{8} + 9x^{6} - 2$	has a lead coefficient of 1 and degree 8
8	has a lead coefficient of 8 and degree 0

A. Polynomials

1) Terms
   a) A polynomial with 1 term is a monomial
   b) A polynomial with 2 terms is a binomial
   c) A polynomial with 3 terms is a trinomial

2) Degrees
   a) A polynomial of degree 0 is a constant polynomial
   b) A polynomial of degree 1 is a linear polynomial
   c) A polynomial of degree 2 is a quadratic polynomial
   d) A polynomial of degree 3 is a cubic polynomial

B. To add or subtract polynomials, add or subtract like terms

1) $(2x^2 + 3x - 5) + (-3x^2 + 2x + 4)$
2) $= (2x^2 - 3x^2) + (3x + 2x) + (-5 + 4)$
3) $=(-x^2 + 5x - 1)$

C. To multiply polynomials, use the distributive property

$$\begin{aligned}(x^2 + 2x - 3)(2x^2 - 3x + 1) &= x^2(2x^2 - 3x + 1) + 2x(2x^2 - 3x + 1) - 3(2x^2 - 3x + 1)\\ &= 2x^4 - 3x^3 + x^2 + 4x^3 - 6x^2 + 2x - 6x^2 + 9x - 3\\ &= 2x^4 + x^3 - 11x^2 + 11x - 3\end{aligned}$$

D. Some special products

1) $(x + y)(x - y) = x^2 - y^2$
2) $(x + y)^2 = x^2 + 2xy + y^2$
3) $(x - y)^2 = x^2 - 2xy + y^2$
4) $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
5) $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

E. Examples of special products

1) $(2x + 3y)(2x - 3y) = (2x)^2 - (3y)^2 = 4x^2 - 9y^2$
2) $(a - 5)^2 = a^2 - 2(a)(5) + 5^2 = a^2 - 10a + 25$
3) $(2x + 4)^3 = (2x)^3 + 3(2x)^2(4) + 3(2x)(4)^2 + 4^3 = 8x^3 + 48x^2 + 96x + 64$

F. To factor a polynomial means to write it as a product of prime polynomials.

1) Some special factorizations
   a) Difference of two perfect squares $x^2 - y^2 = (x+y)(x-y)$

   b) Difference of two perfect cubes $x^3 - y^3 = (x-y)(x^2+xy+y^2)$

   c) Sum of two perfect cubes $x^3 + y^3 = (x+y)(x^2-xy+y^2)$

   d) Perfect square trinomials $x^2+2xy+y^2 = (x+y)^2$
   $x^2-2xy+y^2 = (x-y)^2$

2) A general strategy for factoring:

   a) Factor out the greatest common factor

   b) For a binomial, look for:
      1. Difference of two perfect squares
      2. difference of two perfect cubes
      3. sum of two perfect cubes

   c) For a trinomial, look for:
      1. a perfect square trinomial
      2. try FOIL, grouping, or trial and error

   d) For 4 or more terms, use grouping

   e) Go back and factor each factor completely.

G. Examples

1) $10ax^2 - 40ay^2 = 10a(x^2 - 4y^2)$ *Factoring out the GCF*
   $= 10a(x-2y)(x+2y)$ *Factoring the difference of two perfect squares*

2) $64x^6 - 1 = (8x^3-1)(8x^3+1)$ *Difference of 2 perfect squares*
   $= (2x-1)(4x^2+2x+1)(2x+1)(4x^2-2x+1)$
   Sum and difference of 2 perfect cubes

3) $2x^2 - 20ax + 50a^2 = 2(x^2 - 10ax + 25a^2)$ *Factor out GCF*
   $= 2(x^2 - 2(5ax) + (5a)^2)$ *Rewrite to recognize a perfect square trinomial*
   $= 2(x-5a)^2$ *Factor a perfect square trinomial*

4) $3x^2 - 10x - 8$

- This is not a perfect square trinomial, so try grouping.
- The lead coefficient is 3 and the constant is -8, their product is -24.
- The coefficient of the *x* term is -10.
- Find two numbers that multiply to be -24 and add up to be -10.
- The numbers are -12 and 2.
- Rewrite -10x as (-12x + 2x) then use grouping

$3x^2 - 10x - 8 = 3x^2 - 12x + 2x - 8$ *Rewriting with 4 terms to use grouping*

$= (3x^2 - 12x) + (2x - 8)$ *Grouping the first two terms together and the last two terms together.*

$= 3x(x-4) + 2(x-4)$ *Factoring out the GCF from each group*

$= (x-4)(3x+2)$ *Factoring out the GCF of (x-4)*

5) $x^2 - 9y^2 + 12x + 36 = (x^2 + 12x + 36) - 9y^2$ *Rearranging the terms to group as the difference of 2 perfect squares*.

$= (x+6)^2 - (3y)^2$ *Rewriting as the difference of two perfect squares*

$= (x + 6 + 3y)(x + 6 - 3y)$ *Factoring the difference of two perfect squares*

6) $x^3 - x^2y - xy^2 + y^3 = (x^3 - x^2y) - (xy^2 - y^3)$ *Grouping*

$= x^2(x-y) - y^2(x-y)$ *Factor GCF out of each group*

$= (x-y)(x^2 - y^2)$ *Factor out GCF of (x-y)*

$= (x-y)(x-y)(x+y)$ *Factor the difference of two squares*

6. Rational Expressions

- A quotient of polynomials is called a **rational expression.**
- To simplify, add, subtract, multiply, or divide rational expressions, we use the same rules we use for fractions of real numbers.

A. Examples

1) $\frac{x^2+2x+1}{x^2-1}=\frac{(x+1)(x+1)}{(x+1)(x-1)}$ *Completely factor numerator and denominator*

$=\frac{(x+1)}{(x+1)}\cdot\frac{(x+1)}{(x-1)}$ *Factor out the common factors*

$=1\cdot\frac{(x+1)}{(x-1)}$ *Simplify*

$=\frac{x+1}{x-1}$ *Simplify*

2) $\frac{x^2-6x+9}{x^2-1}\cdot\frac{2x-2}{x-3}=\frac{(x^2-6x+9)(2x-2)}{(x^2-1)(x-3)}$ *Multiplication of fractions*

$=\frac{(x-3)(x-3)(2)(x-1)}{(x-1)(x+1)(x-3)}$ *Factor*

$=\frac{(x-3)}{(x-3)}\cdot\frac{(x-1)}{(x-1)}\cdot\frac{2(x-3)}{(x+1)}$ *Factor out common factors*

$=\frac{2(x-3)}{(x+1)}$ *Simplify*

3) $\frac{x+2}{2x-3}\div\frac{x^2-4}{2x^2-3x}=\frac{x+2}{2x-3}\cdot\frac{2x^2-3x}{x^2-4}$ *Division of fractions*

$=\frac{(x+2)(x)(2x-3)}{(2x-3)(x-2)(x+2)}$

$=\frac{x}{x-2}$

4) $\dfrac{6}{(3x-2)}+\dfrac{5}{x}-\dfrac{2}{x^2(3x-2)}$ *The LCD is* $x^2(3x-2)$

$=\dfrac{6x^2}{x^2(3x-2)}+\dfrac{5x(3x-2)}{x^2(3x-2)}-\dfrac{2}{x^2(3x-2)}$ *Rewrite each fraction with the LCD*

$=\dfrac{6x^2+5x(3x-2)-2}{x^2(3x-2)}$ *Add and subtract the fractions*

$=\dfrac{6x^2+15x^2-10x-2}{x^2(3x-2)}$

$=\dfrac{21x^2-10x-2}{x^2(3x-2)}$ *Simplify*

B. To simplify a complex rational expression, multiply the numerator and denominator by the LCD of all the included fractions.

1) Example:

$$\frac{\dfrac{3}{2x-2}-\dfrac{1}{x+1}}{\dfrac{1}{x-1}+\dfrac{x}{x^2-1}}=\frac{\dfrac{3}{2(x-1)}-\dfrac{1}{(x+1)}}{\dfrac{1}{(x-1)}+\dfrac{x}{(x-1)(x+1)}}\cdot\frac{2(x-1)(x+1)}{2(x-1)(x+1)}$$ *Find LCD*

$$=\frac{\dfrac{3}{2(x-1)}\cdot 2(x-1)(x+1)-\dfrac{1}{(x+1)}\cdot 2(x-1)(x+1)}{\dfrac{1}{(x-1)}\cdot 2(x-1)(x+1)+\dfrac{x}{(x-1)(x+1)}\cdot 2(x-1)(x+1)}$$ *Distribute*

$$=\frac{3(x+1)-2(x-1)}{2(x+1)+x(2)}$$ *Simplify*

$$=\frac{x+5}{4x+2}$$

2) An alternative method is to simplify the numerator into a single fraction, simplify the denominator into a single fraction, then divide the fractions. Try this method as practice and make sure you get the same result as above.

C. Problem Set

Perform the indicated operations. Simplify completely whenever possible

1) $(2x+3)(x-4)+4x(x-2)$

2) $\dfrac{6x^2yz^3-xy^2z}{xyz}$

3) $(x+3y)^3$

4) $\dfrac{x^2-25}{x^3-125}$

5) $\dfrac{2}{3x+1}-\dfrac{9}{9x^2+6x+1}$

6) $\dfrac{\dfrac{a}{b}+\dfrac{b}{a}}{\dfrac{1}{a}-\dfrac{1}{b}}$

7) $\dfrac{\dfrac{1}{x+h}-\dfrac{1}{x}}{h}$

7. Solving Equations

A. To solve a **linear equation,** combine like terms and isolate the variable.

Solve for x: $4x-3=-5x+6$

$4x-3+5x+3=-5x+6+5x+3$ *Combine like terms*

$9x=9$ *Simplify*

$x=1$ *Divide*

Check to verify that x = 1 is a solution!

B. To solve an **equation with fractions**, clear the fractions by multiplying everything by the LCD

Solve for x: $\frac{4}{x+2}+\frac{2}{x-2}=\frac{5x-7}{x^2-4}$ *The LCD is* x^2-4

$$(x^2-4)\left(\frac{4}{x+2}\right)+(x^2-4)\left(\frac{2}{x-2}\right)=(x^2-4)\left(\frac{5x-7}{x^2-4}\right)$$

$$(x-2)(4)+(x+2)(2)=(5x-7)$$

$$4x-8+2x+4=5x-7$$

$$6x-4=5x-7$$

$x=-3$ *Check to verify that x = -3 is a solution!*

C. To solve a **polynomial equation**, first try factoring and using the zero product property.

1) Solve for x: $75x^3+35x^2-10x=0$

$5(15x^2+7x-2)=0$

$5(5x-1)(3x+2)=0$

So by the zero product property we have:

5 = 0 or (5x – 1) = 0 or (3x + 2) = 0

5 = 0 is impossible so $x=\frac{1}{5}$ or $x=-\frac{2}{3}$ *Check to verify the solutions!*

2) Solve for x: $25x^2 = 9$

Using the principal of square roots

$25x^2 = 9$

$\sqrt{25x^2} = \pm\sqrt{9}$

$5x = \pm 3$

$x = \pm \frac{3}{5}$

$x = \frac{3}{5} \quad or \quad x = -\frac{3}{5}$

Using factoring

$25x^2 = 9$

$25x^2 - 9 = 0$

$(5x-3)(5x+3) = 0$

$(5x-3) = 0 \quad or \quad (5x+3) = 0$

Both methods give the same answers.

Check to verify the solutions!
When using the principal of square roots, don't forget the $\pm$

3) Solve for x: $-2x^2 + 6x - 3 = 0$

This is not factorable, so we will use the quadratic formula.

If $a \neq 0$, the solutions of the equation $ax^2 + bx - c = 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For the equation above, a = -2, b = 6, and c = -3 so we get:

$$x = \frac{-6 \pm \sqrt{6^2 - 4(-2)(-3)}}{2(-2)} = \frac{-6 \pm \sqrt{36-24}}{-4} = \frac{-6 \pm \sqrt{12}}{-4} = \frac{-6 \pm 2\sqrt{3}}{-4} = \frac{3 \pm \sqrt{3}}{2}$$

4) Solve for x: $(x-3)^2 = 17$

Using the principal of square roots

$$(x-3)^2 = 17$$

$$\sqrt{(x-3)^2} = \pm\sqrt{17}$$

$$(x-3) = \pm\sqrt{17}$$

$$x = 3 \pm \sqrt{17}$$

Using the quadratic formula

$$(x-3)^2 = 17$$

$$x^2 - 6x + 9 = 17$$

$$x^2 - 6x - 8 = 0$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-8)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36+32}}{2}$$

$$x = \frac{6 \pm \sqrt{68}}{2}$$

$$x = \frac{6 \pm 2\sqrt{17}}{2}$$

$$x = 3 \pm \sqrt{17}$$

Check to verify the solutions!

D. To solve an absolute value equation, isolate the absolute value, the use the following principle: For any <u>positive</u> number *p,* and any algebraic expression *f(x):*

- The solutions of $|f(x)| = p$ are those numbers that satisfy $f(x) = p \text{ or } f(x) = -p$
- The equation $|f(x)| = -p$ has no solution.
- The equation $|f(x)| = 0$ is equivalent to $f(x) = 0$

1) Solve $|2x+5| = 13$, by part (a) we get:

2x + 5 = 13 or 2x + 5 = -13
2x = 8 or 2x = -18
x = 4 or x = -9

Check to verify the solutions!

2) Solve $|2x+5| = -13$, by part (b) this has no solution.

3) Solve $|2x+5|=0$, by part (c) we get

2x + 5 = 0
2x = -5
x = -5/2

Check to verify the solution!

E. Quadratic-like equations

A quadratic-like equation is an equation that has the same properties as a quadratic equation and can therefore be solved using the same methods.

1) Solve for x: $x^4 - 5x^2 + 4 = 0$

By making the substitutions $\begin{matrix} u = x^2 \\ u^2 = x^4 \end{matrix}$ we get the following quadratic equation

$$u^2 - 5u + 4 = 0$$
$$(u-4)(u-1) = 0$$
$$u-4=0 \ or \ u-1=0$$

u = 4 or u = 1

Substituting back for *u* we get

$$x^2 = 4 \ or \ x^2 = 1$$
$$x = 2 \ or \ x = -2 \ or \ x = 1 \ or \ x = -1$$

2) Solve for x: $x^{-2} - 2x^{-1} - 5 = 0$

By making the substitutions $\begin{matrix} u = x^{-1} \\ u^2 = x^{-2} \end{matrix}$ we get the following quadratic equation

$$u^2 - 2u - 5 = 0$$

$$u = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$$

$$u = \frac{2 \pm \sqrt{4+20}}{2}$$

$$u = \frac{2 \pm \sqrt{24}}{2}$$

$$u = \frac{2 \pm 2\sqrt{6}}{2}$$

$u = 1 \pm \sqrt{6}$

Substituting back for *u* we get

$x^{-1} = 1 + \sqrt{6}\ or\ x^{-1} = 1 - \sqrt{6}$

so $x = \dfrac{1}{1+\sqrt{6}} = \dfrac{1-\sqrt{6}}{-5}$ or $x = \dfrac{1}{1-\sqrt{6}} = \dfrac{1+\sqrt{6}}{-5}$

Check to verify the solutions!

F. Problem Set

Solve the following equations. Check all your answers!

1) $(x+5)^2 + 3 = (x-2)^2$

2) $\dfrac{2}{2x+5} + \dfrac{3}{2x-5} = \dfrac{10x+5}{4x^2-25}$

3) $48x^2 + 12x - 90 = 0$

4) $\dfrac{3x}{x-2} + \dfrac{1}{x+2} = \dfrac{-4}{x^2-4}$

5) $16x^2 = 49$

6) $(x+4)^2 = 31$

7) $x^2 - 6x - 3 = 0$

8) $2|5x+2| - 1 = 5$

9) $4x^4 + 10x^3 = 6x^2 + 15x$ *Hint: think grouping*

10) $2x - 3\sqrt{x} + 1 = 0$