Regular Geometry and XL style questions

Example 1 and 2 are the types of problems we'd expect both Geometry and Geometry XL students to be able to do. Example 3 is a Geometry XL type of problem. We have worked through the problems to show what our expectation is of our students regarding clearly showing all the steps in completing a problem.

Equations of Circles

The standard equation of a circle with radius **r** and center **(h, k)** is:
$(x - h)^2 + (y - k)^2 = r^2$

Example 1: Write the standard equation of the circle whose center is (-2, 3) and whose radius is 4.

Solution:
$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - (-2))^2 + (y - 3)^2 = 4^2$$
$$(x + 2)^2 + (y - 3)^2 = 16$$

Example 2: Write the standard equation of the circle whose center is (1, 1) and passes through the point (-1, 4).

Solution:
The radius is the distance from (-1, 4) to the center (1, 1)

$$r = \sqrt{(-1 - 1)^2 + (4 - 1)^2} = \sqrt{(-2)^2 + (3)^2} = \sqrt{13}$$

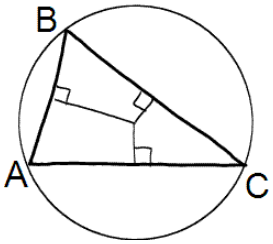
Thus, the equation is: $(x - 1)^2 + (y - 1)^2 = \left(\sqrt{13}\right)^2$

$$(x - 1)^2 + (y - 1)^2 = 13$$

Example 3: A circle passes through the points A(-1,5), B(7, 1), and C(5, -3). Find the equation of the circle.

Review (reminder):

Circumcenter = common point of perpendicular bisectors of a triangle. It is equidistant from all 3 vertices of a triangle.



1) Find midpoint of AB: $\left(\frac{-1+7}{2}, \frac{5+1}{2}\right) = (3,3)$

2) Find slope of AB: $\frac{5-1}{-1-7} = \frac{4}{-8} = -\frac{1}{2}$

3) Slope of $\perp$ bisector of AB = 2

4) Equation of $\perp$ bisector: y = 2x + b
3 = 2(3) + b
b = -3
y=2x-3

Do the same for BC (or AC):

5) Midpoint of BC = $\left(\frac{7+5}{2}, \frac{1+(-3)}{2}\right) = (6,-1)$

6) Slope of BC = $\frac{1-(-3)}{7-5} = \frac{4}{2} = 2$

7) Slope of $\perp$ bisector of BC = -1/2

8) Equation of $\perp$ bisector: y = -(1/2)x+b
-1 = -(1/2)6 + b
b = 2
y = -(1/2)x+2

9) Circumcenter is where the two bisectors meet:

$2x-3 = -\frac{1}{2}x+2$

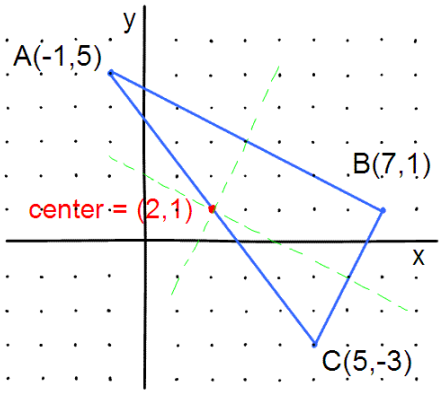
$\frac{5}{2}x = 5$

x = 2

10) If x = 2, then y = 2x – 3
y = 2(2) – 3 = 1

11) Center of circle = (2,1)

12) Radius = $\sqrt{(7-2)^2+(1-1)^2} = \sqrt{25} = 5$



Answer: $(x-2)^2+(y-1)^2 = 25$