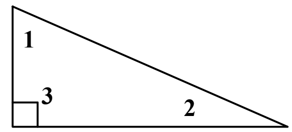
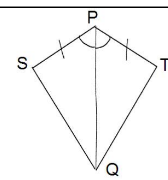
Geometry Sample Problems

Sample Proofs – Below are examples of some typical proofs covered in Jesuit Geometry classes. Shown first are blank proofs that can be used as sample problems, with the solutions shown second.

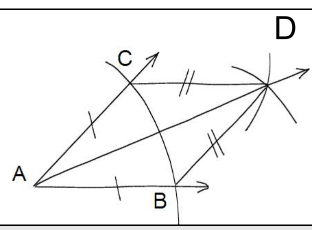
Proof #1

Given: a triangle with m∠3 = 90° Prove: ∠1 and ∠2 are complementary	
Statements	**Reasons**
1. m∠3 = 90°	1. Given

Proof #2

Given: PQ bisects ∠SPT, SP ≅ PT Prove: ΔSPQ ≅ ΔTPQ	
Statements	**Reasons**
1. PQ bisects ∠SPT, SP ≅ PT	1. Given

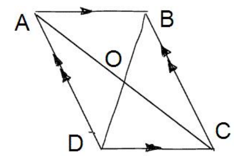
Proof #3

	Given: AB ≅ AC, BD ≅ CD Prove: AD bisects ∠CAB
Statements	**Reasons**
1. AB ≅ AC, BD ≅ CD	1. Given

Proof #4

Given:
p’gram ABCD w/ diagonals AC & BD

Prove: AO ≅ OC and DO ≅ OB

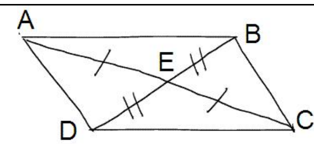


Statements	Reasons
1. p’gram ABCD w/ diagonals AC & BD	1. Given

Proof #5

Given: AE ≅ EC, DE ≅ EB

Prove: ABCD is a p’gram

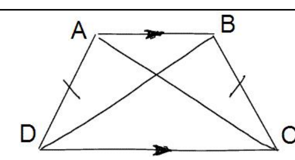


Statements	Reasons
1. AE ≅ EC, DE ≅ EB	1. Given
	Defn of a p’gram
ABCD is a p’gram	

Proof #6

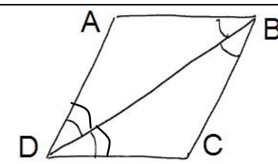
Given: trapezoid ABCD
AD ≅ BC

Prove: AC ≅ BD



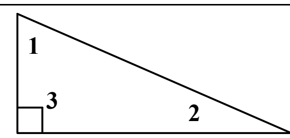
Statements	Reasons

Proof #7

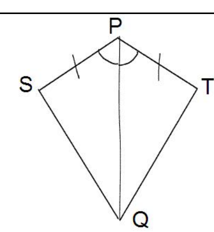
Given: p'gram ABCD ∠CBD ≅ ∠ABD, ∠BDC ≅ ∠BDA Prove: ABCD is a rhombus	
Statements	**Reasons**

Solutions

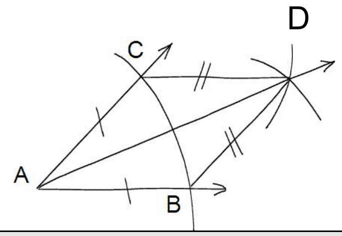
Proof #1

Given: a triangle with m∠3 = 90° Prove: ∠1 and ∠2 are complementary	
Statements	**Reasons**
1. m∠3 = 90°	1. Given
2. m∠1 + m∠2 + m∠3 = 180°	2. Sum of ∠'s for a Δ
3. m∠1 + m∠2 + 90°= 180°	3. Substitution
4. m∠1 + m∠2 = 90°	4. Subt. Prop. of equality
5. ∠1 and ∠2 are complementary	5. Defn of comp. ∠'s

Proof #2

Given: $\overline{PQ}$ bisects ∠SPT, $\overline{SP} \cong \overline{PT}$ Prove: ΔSPQ ≅ ΔTPQ	
Statements	**Reasons**
1. $\overline{PQ}$ bisects ∠SPT	1. Given
2. $\overline{SP} \cong \overline{PT}$	2. Given
3. ∠SPQ ≅ ∠QPT	3. Defn of ∠ bisector
4. $\overline{PQ} \cong \overline{PQ}$	4. Reflexive prop. of congruence
5. ΔSPQ ≅ ΔTPQ	5. SAS congruence postulate

Proof #3



Given: $\overline{AB} \cong \overline{AC}$, $\overline{BD} \cong \overline{CD}$

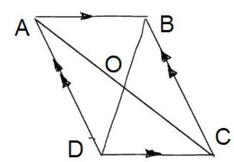
Prove: $\overline{AD}$ bisects $\angle$CAB

Statements	Reasons
1. $\overline{AB} \cong \overline{AC}$, $\overline{BD} \cong \overline{CD}$	1. Given
2. $\overline{AD} \cong \overline{AD}$	2. Reflexive prop. of congruence
3. ΔACD $\cong$ ΔABD	3. SSS congruence postulate
4. $\angle$CAD $\cong$ $\angle$BAD	4. CPCTC
5. $\overline{AD}$ bisects $\angle$CAB	5. Defn of angle bisector

Proof #4

Given: p'gram ABCD w/ diagonals $\overline{AC}$ & $\overline{BD}$

Prove: $\overline{AO} \cong \overline{OC}$ and $\overline{DO} \cong \overline{OB}$

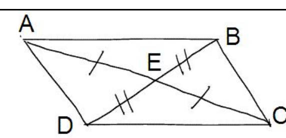


Statements	Reasons
1. p'gram ABCD w/ diagonals $\overline{AC}$ & $\overline{BD}$	1. Given
2. $\overline{AB} \parallel \overline{DC}$	2. Defn of parallelogram
3. $\angle$BAO $\cong$ $\angle$DCO and $\angle$ABO $\cong$ $\angle$CDO	3. Alt Int $\angle$'s $\cong$
4. $\overline{AB} \cong \overline{CD}$	4. Opposite sides of a p'gram $\cong$
5. ΔABO $\cong$ ΔDCO	5. ASA congruence theorem
6. $\overline{AO} \cong \overline{OC}$ and $\overline{DO} \cong \overline{OB}$	6. CPCTC

Proof #5

Given: $\overline{AE} \cong \overline{EC}$ and $\overline{DE} \cong \overline{EB}$

Prove: ABCD is a p'gram

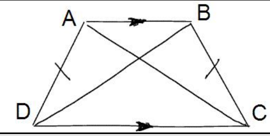


Statements	Reasons
1. $\overline{AE} \cong \overline{EC}$ and $\overline{DE} \cong \overline{EB}$	1. Given
2. $\angle$BEC $\cong$ $\angle$AED	2. Vertical $\angle$'s $\cong$
3. ΔBEC $\cong$ ΔAED	3. SAS congruence postulate
4. $\angle$CBE $\cong$ $\angle$ADE	4. CPCTC
5. $\overline{BC} \parallel \overline{AD}$	5. Alt int $\angle$'s $\cong$ $\rightarrow$ lines $\parallel$
6. $\angle$AEB $\cong$ $\angle$CED	6. Vertical $\angle$'s $\cong$
7. ΔAEB $\cong$ ΔCED	7. SAS congruence postulate
8. $\angle$DCE $\cong$ $\angle$BAE	8. CPCTC
9. $\overline{AB} \parallel \overline{DC}$	9. Alt int $\angle$'s $\cong$ $\rightarrow$ lines $\parallel$
10. ABCD is a p'gram	10. Defn of a parallelogram

Proof #6

Given: trapezoid ABCD with $\overline{AD} \cong \overline{BC}$

Prove: $\overline{AC} \cong \overline{BD}$

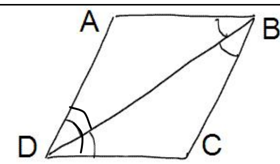


Statements	Reasons
1. Trapezoid ABCD, $\overline{AD} \cong \overline{BC}$	1. Given
2. ABCD is isosceles Trapezoid	2. Definition of isosceles trapezoid
3. $\overline{DC} \cong \overline{DC}$	3. Reflexive prop of ≅
4. ∠BCD ≅ ∠ADC	4. Base ∠'s in an isosceles trapezoid are ≅
5. ΔBCD ≅ ΔADC	5. SAS
6. $\overline{AC} \cong \overline{BD}$	6. CPCTC

Proof #7

Given: p'gram ABCD
∠CBD ≅ ∠ABD, ∠BDC ≅ ∠BDA

Prove: ABCD is a rhombus

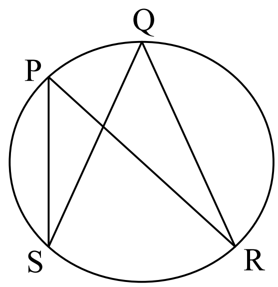


Statements	Reasons
1. p'gram ABCD	1. Given
2. ∠CBD ≅ ∠ABD, ∠BDC ≅ ∠BDA	2. Given
3. $\overline{BD} \cong \overline{BD}$	3. Reflexive prop of ≅
4. ΔBCD ≅ ΔBAD	4. ASA congruence theorem
5. $\overline{CD} \cong \overline{AD}$	5. CPCTC
6. $\overline{BC} \cong \overline{AB}$	6. CPCTC
7. $\overline{CD} \cong \overline{AB}$, $\overline{AD} \cong \overline{BC}$	7. Opposite sides of parallelogram are congruent
8. $\overline{AB} \cong \overline{BC} \cong \overline{AD} \cong \overline{CD}$	8. Transitive prop of ≅
9. ABCD is a rhombus	9. Defn of a rhombus

Worked sample problems from the Geometry Challenge Exam

The following are samples of the type of questions that will appear on the Challenge Exam and the way the solutions should be written out. This is not meant to be comprehensive, but to give an idea of what a well thought out and written response would look like.

Example 1:



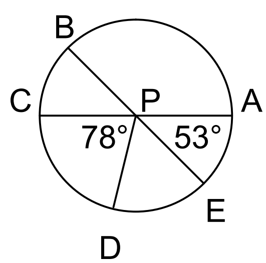
Find $m\angle PSQ$
if $m\angle PSQ = 3y + 4$ and $m\angle PRQ = 2y + 16$

Answer:
Since $\angle PSQ$ is an inscribed angle, then
$m\angle PSQ = \frac{1}{2} marcPQ$
Similarly, $m\angle PRQ = \frac{1}{2} marcPQ$

Therefore $m\angle PRQ = m\angle PSQ$
Substituting, 2y + 16 = 3y + 4
Then y = 12

And $m\angle PSQ = 3y + 4 = 3(12) + 4 = 40$

Example 2:

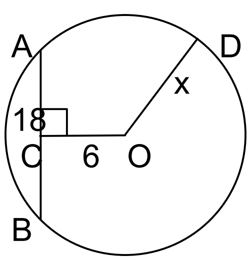


Find the measure of arc ADB.

Answer:
Arc ADC is a semicircle; therefore, the measure of arc ADC = 180º
The measure of $m\angle BPC = 53°$ (vertical angles)

Therefore, the measure of
arc ADB = 53º + 180º = 233º

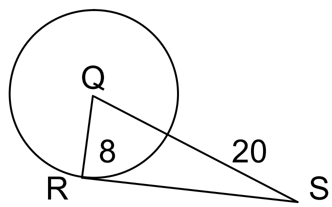
Example 3:



Find the value of x. AB=18

Answer:
OC is perpendicular to chord AB, therefore, OC bisects chord AB.
Draw a line from A to O. AOC forms a right triangle.
Use the Pythagorean theorem to find hypotenuse OA.
$6^2 + 9^2 = (OA)^2$
OA = $\sqrt{117}$
Since OA and OD are both radii, x = $\sqrt{117}$

Example 4:



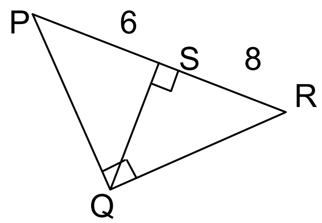
$\overline{SR}$ is tangent to circle Q at R.
Find RS. (QS = 20, QR = 8)

Answer:
The radius, QR, is perpendicular to RS at the tangent point R.
Therefore, $\angle$QRS is a right angle.

Use the Pythagorean theorem to find "leg" RS.
$8^2 + (RS)^2 = 20^2$; $336 = (RS)^2$
RS = $4\sqrt{21}$

Example 5:

Given: PS = 6, SR = 8, find the value of SQ.



Answer:
Since SQ is an altitude of triangle PQR, SQ is the geometric mean of segments PS and SR.
Therefore we have the ratio $\frac{PS}{SQ} = \frac{SQ}{SR}$

$\frac{6}{SQ} = \frac{SQ}{8}$

$(SQ)^2 = 48$

$SQ = 4\sqrt{3}$

<u>Regular Geometry and XL style questions</u>

Example 1 and 2 are the types of problems Jesuit expects both Geometry and Geometry XL students to be able to do. Example 3 is a Geometry XL type of problem. The problems show our expectations of students regarding all the steps necessary to complete a problem.

<u>Equations of Circles</u>

The standard equation of a circle with radius **r** and center **(h, k)** is: $(x - h)^2 + (y - k)^2 = r^2$

<u>Example 1</u>: Write the standard equation of the circle whose center is (-2, 3) and whose radius is 4.

<u>Solution</u>:

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - (-2))^2 + (y - 3)^2 = 4^2$$
$$(x + 2)^2 + (y - 3)^2 = 16$$

<u>Example 2</u>: Write the standard equation of the circle whose center is (1, 1) and passes through the point (-1, 4).

<u>Solution</u>: The radius is the distance from (-1, 4) to the center (1, 1).

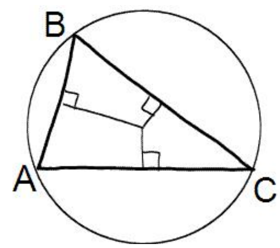
$$r = \sqrt{(-1 - 1)^2 + (4 - 1)^2} = \sqrt{(-2)^2 + (3)^2} = \sqrt{13}$$

Thus, the equation is: $(x - 1)^2 + (y - 1)^2 = \left(\sqrt{13}\right)^2$ or $(x - 1)^2 + (y - 1)^2 = 13$

__Example 3__: A circle passes through the points A(-1,5), B(7, 1), and C(5, -3). Find the equation of the circle.

Review:

Circumcenter = common point of perpendicular bisectors of a triangle. It is equidistant from all 3 vertices of a triangle.



1) Find midpoint of AB: $\left(\frac{-1+7}{2},\frac{5+1}{2}\right)=(3,3)$
2) Find slope of AB: $\frac{5-1}{-1-7}=\frac{4}{-8}=-\frac{1}{2}$
3) Slope of ⊥ bisector of AB = 2
4) Equation of ⊥ bisector: y = 2x + b
   3 = 2(3) + b
   b = -3
   y = 2x – 3

Do the same for BC (or AC):

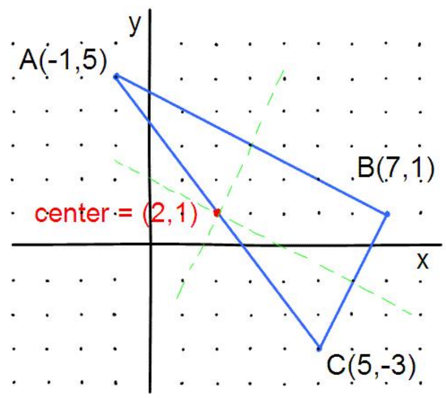
5) Midpoint of BC = $\left(\frac{7+5}{2},\frac{1+(-3)}{2}\right)=(6,-1)$
6) Slope of BC = $\frac{1-(-3)}{7-5}=\frac{4}{2}=2$
7) Slope of ⊥ bisector of BC = -1/2
8) Equation of ⊥ bisector: y = -(1/2)x+b
   -1 = -(1/2)6 + b
   b = 2
   y = -(1/2)x + 2
9) Circumcenter is where the two bisectors meet:

$$2x-3=-\frac{1}{2}x+2$$

$$\frac{5}{2}x=5$$

x = 2

10) If x = 2, then y = 2x – 3
    y = 2(2) – 3 = 1
11) Center of circle = (2,1)
12) Radius = $\sqrt{(7-2)^2+(1-1)^2}=\sqrt{25}=5$



Answer: $(x-2)^2+(y-1)^2=25$