

# **FUNCTIONS, STATISTICS, and TRIGONOMETRY**

**GOALS:** Students will develop skills and understanding in:

1. Exploring Data
2. Functions and Models
3. Transformations of Graphs and Data
4. Circular Functions
5. Trigonometric Functions
6. Root, Power, and Logarithm Functions
7. Probability and Simulation
8. Sequences, Series, and Combinations
9. Polynomial Functions
10. Binomial and Normal Distributions
11. Matrices and Trigonometry
12. Quadratic Relations
13. Further work with Trigonometry

**OBJECTIVES:** Students will be able to:

*(Parentheticals refer to Elementary Statistics: “A Step by Step Approach”, Alan G. Bluman., 2004)*

## **Goal 1. Exploring Data**

- 1.1. Use samples to make inferences about populations (1-1)
- 1.2. Determine relationships and interpret data presented in a table (1-1)
- 1.3. Draw graphs to display data (1-1, 1-2, 1-4, 1-5, 1-6)
- 1.4. Calculate measures of center and spread for data sets (1-2 to 1-4, 1-7)
- 1.5. Use statistics to describe data sets and to compare or contrast data sets (1-2, 1-3, 1-4, 1-7)
- 1.6. Use  $\Sigma$ -notation to represent a sum, mean, variance, or standard deviation (1-3, 1-7)
- 1.7. Compare measures of center and measures of spread (1-3, 1-7)
- 1.8. Read and interpret box plots (1-4)
- 1.9. Read and interpret dotplots and histograms (1-2, 1-5)
- 1.10. Read and interpret bar graphs, circle graphs, and coordinate graphs (1-1, 1-6)

## **Goal 2. Functions and Models**

- 2.1. Evaluate functions described with Euler's notation (2-1, 2-7)
- 2.2. Identify the independent and dependent variables, domain, and range of a function (2-1, 2-4, 2-6, 2-7)
- 2.3. Interpret properties of relations from graphs (2-1, 2-4, 2-6, 2-7)

- 2.4. Identify properties of regression lines and of the correlation coefficient (2-2, 2-3)
- 2.5. Find and interpret linear, exponential, and quadratic models (2-2, 2-3, 2-4, 2-5, 2-6)
- 2.6. Graph linear, exponential, quadratic, and step functions (2-2, 2-4, 2-6, 2-7)
- 2.7. Use scatterplots to draw conclusions about models for data (2-2, 2-3, 2-5, 2-6, 2-8)
- 2.8. Describe properties of quadratic and exponential functions (2-4, 2-6)
- 2.9. Use step functions to model situations (2-7)

### **Goal 3. Transformations of Graphs and Data**

- 3.1. Represent and graph parent functions (3-1)
- 3.2. Use the Graph-Translation Theorem or the Graph Scale-Change Theorem to find transformation images (3-2, 3-5)
- 3.3. Apply the Graph-Translation Theorem or the Graph Scale-Change Theorem to make or identify graphs (3-2, 3-5)
- 3.4. Describe the effects of translations or scale changes on functions and their graphs (3-2, 3-5)
- 3.5. Describe the effects of translations or scale changes on measures of center or spread (3-3, 3-6)
- 3.6. Use translations, scale changes, or z-scores to analyze data (3-3, 3-6, 3-9)
- 3.7. Describe the symmetries of graphs (3-4)
- 3.8. Identify properties of composites and inverses (3-7, 3-8)
- 3.9. Find formulas and values of composite functions (3-7)
- 3.10. Find and graph inverses of functions (3-8)
- 3.11. Identify properties of z-scores (3-9)
- 3.12. From the graph of a function, determine its symmetries or whether its inverse is a function (3-4, 3-8)

### **Goal 4. Circular functions**

- 4.1. Convert between degrees, radians, and revolutions (4-1)
- 4.2. Find lengths of circular arcs and areas of sectors (4-2)
- 4.3. Solve problems involving lengths of arcs or areas of sectors (4-2)
- 4.4. Find sines, cosines, and tangents of angles from a calculator and the unit circle (4-3, 4-5)
- 4.5. Apply the definitions of the sine, cosine, and tangent functions (4-3, 4-4, 4-6)
- 4.6. Apply theorems about sines, cosines, and tangents (4-4, 4-5)
- 4.7. Identify the amplitude, period, frequency, phase shift, and other properties of circular functions (4-7, 4-8, 4-9)

- 4.8. Find and use equations of circular functions to solve problems about periodic phenomena (4-7, 4-8, 4-9, 4-10)
- 4.9. Draw, interpret, or state equations for graphs of circular functions (4-6, 4-7, 4-8, 4-9)
- 4.10. Find or graph transformations of circular functions (4-7, 4-8, 4-9)

**Goal 5. Trigonometric Functions**

- 5.1. Find sines, cosines, and tangents of angles in a right triangle (5-1)
- 5.2. Use trigonometry to find lengths, areas, or angle measures (5-1, 5-2, 5-4)
- 5.3. Interpret the Law of Sines, Law of Cosines and related theorems (5-2, 5-4)
- 5.4. Solve problems using trigonometric ratios and the Laws of Sines and Cosines (5-1, 5-2, 5-4)
- 5.5. Solve trigonometric equations (5-3, 5-5, 5-6, 5-7)
- 5.6. State properties of, graph, identify, and evaluate inverse trigonometric functions (5-3, 5-5, 5-6)
- 5.7. Write and solve equations for phenomena described by trigonometric and circular functions (5-3, 5-4, 5-5, 5-6, 5-7)

**Goal 6. Root, Power, and Logarithm Functions**

- 6.1. Evaluate rational exponential expressions (6-1, 6-2)
- 6.2. Use rational exponents to model situations (6-1, 6-2)
- 6.3. Evaluate logarithms (6-3, 6-4, 6-5, 6-6)
- 6.4. Use properties of logarithms (6-5, 6-6, 6-7)
- 6.5. Describe properties of rational power,  $n$ th root, and logarithm functions (6-1, 6-2, 6-3, 6-4, 6-5)
- 6.6. Graph and interpret graphs of  $n$ th root, rational power, and logarithm functions (6-1, 6-2, 6-3, 6-4)
- 6.7. Solve exponential equations (6-6)
- 6.8. Solve problems arising from exponential or logarithmic models (6-4, 6-6, 6-7)

**Goal 7. Probability and Simulation**

- 7.1. List sample spaces and events for probabilistic experiments (7-1)
- 7.2. Compute probabilities (7-1, 7-2, 7-3)
- 7.3. Determine whether events are mutually exclusive, independent, or complementary (7-2, 7-5)
- 7.4. State and use properties of probabilities (7-1, 7-2, 7-5)
- 7.5. Apply probabilities to real situations (7-1, 7-2, 7-5)
- 7.6. Evaluate expressions using factorials (7-3, 7-4)
- 7.7. Use counting principles and theorems to find the number of ways of arranging objects (7-3, 7-4)

- 7.8. Solve equations using factorials (7-4)
- 7.9. Construct, graph, and interpret probability distributions (7-6)
- 7.10. Design and conduct simulations with and without technology (7-7, 7-8)

**Goal 8. Sequences, Series, and Combinations**

- 8.1. Find terms of sequences from explicit or recursive formulas (8-1)
- 8.2. Find explicit or recursive formulas for the  $n$ th term of an arithmetic or geometric sequence (8-1)
- 8.3. Determine whether a sequence is arithmetic or geometric. (8-1)
- 8.4. Solve problems involving arithmetic and geometric sequences and series (8-1, 8-3, 8-4, 8-5)
- 8.5. Evaluate arithmetic or geometric series (8-3, 8-4)
- 8.6. Determine limits of sequences and series (8-2, 8-5)
- 8.7. Prove and apply properties of combinations (8-6, 8-7)
- 8.8. Identify properties represented by the patterns in Pascal's Triangle (8-7)
- 8.9. Expand binomials (8-8)
- 8.10. Determine probabilities in situations involving binomial experiments (8-9)

**Goal 9. Polynomial Functions**

- 9.1. Construct and interpret polynomials that model real situations (9-1, 9-3)
- 9.2. Calculate or approximate zeros and relative extrema of polynomial functions (9-2)
- 9.3. Determine an equation for a polynomial function from data points (9-3)
- 9.4. Relate properties of polynomial functions and their graphs (9-2, 9-5, 9-7)
- 9.5. Divide polynomials (9-4)
- 9.6. Apply the Remainder Theorem, Factor Theorem, and Factor-Solution-Intercept Equivalence Theorem (9-4, 9-5)
- 9.7. Solve polynomial equations (9-5, 9-6, 9-8, 9-9)
- 9.8. Perform operations with complex numbers (9-6)
- 9.9. Apply the Fundamental Theorem of Algebra and Conjugate Zeros Theorem (9-7)

**Goal 10. Binomial and Normal Distributions**

- 10.1. Graph, interpret, compare, and contrast different binomial probability distribution graphs (10-1)
- 10.2. Solve probability problems using binomial or normal distributions (10-1, 10-2, 10-6)

- 10.3. Calculate the mean and standard deviation of a binomial probability distribution (10-2)
- 10.4. Use binomial and normal distributions to test hypotheses (10-3, 10-7)
- 10.5. Graph, interpret, and use properties of normal distributions (10-4, 10-5, 10-6)
- 10.6. Apply the Central Limit Theorem (10-7)
- 10.7. Apply confidence intervals to real-world problems (10-8)

### **Goal 11. Matrices and Trigonometry**

- 11.1. Apply properties of matrices and matrix multiplication (11-1)
- 11.2. Represent reflections, rotations, scale changes, size changes, and composites of transformations as matrices and matrix products (11-2, 11-3, 11-4)
- 11.3. Apply the Addition and Double Angle Formulas (11-5, 11-6)
- 11.4. Use matrices to solve systems of equations (11-7)
- 11.5. Find the inverse of a  $2 \times 2$  matrix (11-7)

### **Goal 12. Quadratic Relations**

- 12.1. State and apply properties of ellipses and hyperbolas to write equations given their graphs, or graph them given their equations (12-1, 12-2, 12-3, 12-4)
- 12.2. Describe the intersections of a plane and a cone of 2 nappes (12-1, 12-5)
- 12.3. Determine information about elliptical orbits (12-2)
- 12.4. Graph transformation images of ellipses and hyperbolas (12-2, 12-3, 12-4, 12-5)
- 12.5. Rewrite equations of conic sections in the general form of a quadratic relation in two variables (12-5)
- 12.6. Describe graphs of quadratic equations (12-5)

### **Goal 13. Further work with Trigonometry**

- 13.1. Evaluate and apply properties of the reciprocal trigonometric functions (13-1)
- 13.2. Use an automatic grapher to test proposed identities. Prove identities (13-2, 13-3)
- 13.3. Describe singularities of functions (13-3)
- 13.4. Given rectangular coordinates of a point, determine and graph its polar coordinates and vice versa (13-4)
- 13.5. Graph and interpret graphs of polar equations (13-5)
- 13.6. Graph and represent complex numbers in different forms (13-6, 13-7)
- 13.7. Find roots and powers of complex numbers (13-8)