HPC Summer Math 2025

Do all work on other paper. Show all support work. Express intercepts as ordered pairs. There will be a summer math test during the first week of school.

TOPIC 1

The x-intercept of a function can always be calculated by setting y equal to zero, creating an x-intercept of (x,0); Similarly, the y-intercept of a function can always be calculated by setting x equal to zero, creating a y-intercept of (0,y). Functions can have at most one y-intercept.

Find the x- and y-intercepts of:

1) $2x - 3y = 12$

2) $y = -\dfrac{1}{2}x - 7$

3) $x^2 - 2y = 16$

4) $g(x) = 4 - |2x + 6|$

5) $f(x) = 2\sqrt{x + 4}$

6) $y = 2(3)^x - 18$

TOPIC 2

The slope of a line can be calculated as $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. Lines have a slope because their rate of change is constant. Lines with positive slopes rise to the right; lines with negative slopes fall to the right.

Find the slope of the line that contains:

7) (-2,5) and (6,-7)

8) (3,-1) and (-2,-5)

9) (3,7) and (-4,7)

10) (5,-2) and (5,3)

TOPIC 3

Vertical lines have an undefined slope and are of the form x = a, where a is a constant. Horizontal lines have a slope of 0 and are of the form y = b, where b is a constant.

Give the slope of the following lines, then graph on an x/y plane:

11) $y = -2$

12) $x = 3$

TOPIC 4

There are three forms of the equation of a line:

- Slope-intercept form $y = mx + b$ [where m is the slope and b is the y-intercept]
- Point-slope form $y - y_1 = m(x - x_1)$ [where m is the slope and (x_1, y_1) is a point on the line]
- Standard form $Ax + By = C$ [where A, B and C are real numbers and A and B are not both zero]

Give the slope and y-intercept, then graph on an x/y plane:

13) $y = 2x - 5$

14) $y = -\dfrac{3}{2}x + 1$

15) $3x + 4y = 8$

16) $y - 3 = 2(x + 1)$

Graph using intercepts (find the x- and y-intercepts, then graph and connect them):

17) $2x - 5y = 10$

18) $3x + 2y = 9$

TOPIC 5

Parallel (//) lines have equal slopes and do not intersect each other. Perpendicular (⊥) lines have opposite reciprocal slopes (ie their slopes multiply to -1) and they intersect each other at 90° (right) angles.

Find the slope of the line that:

19) is // to $y = \dfrac{2}{3}x - 11.25$

20) // to $y + 2 = -3(x - 5)$

21) is ⊥ to $3y + 2x = 1$

22) is ⊥ to the line that contains (-4,1) and (7,-1)

TOPIC 6

Equations of lines can be written if you are given the slope and one point on the line, if you are given two points on the line, or if you are given a point on the line and the slope (or equation) of a parallel or perpendicular line. Determine the slope first, then write the equation in point-slope form. From there you can rearrange into slope-intercept form or standard form as necessary.

Write the equation of the line, in slope-intercept form, that:

23) contains (3,-2) and is // to $y = \dfrac{1}{2}x + 7$

24) contains (-2,9) and (0,6)

25) contains (-4,5) and is ⊥ to $3x - y = 5$
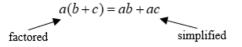
26) contains (-1,3) and (2,8)

TOPIC 7

Polynomial expressions can be simplified by multiplying to remove any parentheses and then adding like terms. In simplified form, expressions may not contain any parentheses. Simplified polynomials are written in standard form (in order of decreasing degree).

Simplify:

27) $2x(x - 2) - (3 + x) + 5(x^2 + 7x)$

28) $(x - 2)(x + 10)$

29) $(3x + 5)(2x - 3)$

30) $(x - 5)^2$

31) $4 - (x + 3)^2$

32) $(2x + 1)^2 - (x + 3)(x - 5) + (2x - 3)(x - 7) - (x - 2)(x + 2)$

TOPIC 8

Some polynomials can be factored by rewriting their terms as a product of factors. Always begin by factoring out the greatest common factor (GCF). The process of factoring (adding parentheses) is the opposite of the process of simplifying (eliminating parentheses)

$$a(b + c) = ab + ac$$

factored   simplified

Factor completely:

33) $3a^2b - 6ab^3 + 15ab$

34) $x^2 + 5x$

35) $x^2 - 5x - 50$

36) $2x^2 + 34x + 144$

37) $2x^2 - 7x - 15$

38) $6x^2 - 19x - 7$

39) $x^3 - 3x^2 + 2x - 6$

40) $6x^4 + 15x^3 - 18x^2 - 45x$

TOPIC 9

The following are useful factoring forms:

- Difference of two squares $a^2 - b^2 = (a+b)(a-b)$
- Perfect square trinomials $a^2 + 2ab + b^2 = (a+b)^2$ and $a^2 - 2ab + b^2 = (a-b)^2$
- Difference of two cubes $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- Sum of two cubes $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Factor completely:

41) $9a^2 - 25b^2$ 42) $2x^2 - 2$ 43) $b^4 - 16$

44) $x^2 + 8x + 16$ 45) $4x^2 - 20x + 25$ 46) $2ax^2 - 12ax + 18a$

47) $x^3 - 125$ 48) $32x^3 + 4$

TOPIC 10

You can complete the square for any trinomial with a lead coefficient of one to create a perfect square trinomial, which can then factor as a binomial square. Complete the square by adding $\left(\frac{1}{2}b\right)^2$ to the trinomial $x^2 + bx + c$.

Complete the square, then write the resulting binomial square:

49) $x^2 + 12x +$ ____   50) $a^2 - 20a +$ ____   51) $x^2 + 7x +$ ____

TOPIC 11

Functions can be evaluated by inputting a number or expression for the independent variable (x). If you input a number, your output will be a number. This creates an ordered pair that identifies a point on the graph of the function. Any point on the graph will make the function rule true and any point that makes the function rule true is a point on the graph.

Evaluate:

52) If $f(x) = 2x + 10$, find $f(-3)$ 53) If $g(x) = 2x^2 - x + 4$, find $g(3)$

54) If $h(x) = -2\sqrt{x-5} + 3$, find $h(21)$ 55) If $j(x) = 2x^3 - 3x^2 + 6$, find $j(-1)$

Determine if:

56) (0,3) is on the graph of $f(x) = 2x - 3$ 57) (-2,13) is on the graph of $f(x) = x^2 - 2x + 5$

TOPIC 12

There are two domain restrictions you should be familiar with:

- The denominator of a fraction cannot equal zero   [fractions with a denominator of zero are undefined]
- The radicand of an even-indexed radical cannot be negative   [even-indexed radicals with negative radicands represent imaginary numbers]

State the domain for:

58) $f(x) = \dfrac{x+1}{x-7}$ 59) $g(x) = \dfrac{5x}{x^2 - x - 30}$ 60) $h(x) = -\sqrt{3x-12} - 7$